Algebraic Reasoning (IMRA)

Subject: Mathematics
Grade: 10
Expectations: 34
Breakouts: 175

(a) Introduction.

1. The desire to achieve educational excellence is the driving force behind the Texas essential knowledge and skills for mathematics, guided by the college and career readiness standards. By embedding statistics, probability, and finance, while focusing on fluency and solid understanding, Texas will lead the way in mathematics education and prepare all Texas students for the challenges they will face in the 21st century.
2. The process standards describe ways in which students are expected to engage in the content. The placement of the process standards at the beginning of the knowledge and skills listed for each grade and course is intentional. The process standards weave the other knowledge and skills together so that students may be successful problem solvers and use mathematics efficiently and effectively in daily life. The process standards are integrated at every grade level and course. When possible, students will apply mathematics to problems arising in everyday life, society, and the workplace. Students will use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution. Students will select appropriate tools such as real objects, manipulatives, paper and pencil, and technology and techniques such as mental math, estimation, and number sense to solve problems. Students will effectively communicate mathematical ideas, reasoning, and their implications using multiple representations such as symbols, diagrams, graphs, and language. Students will use mathematical relationships to generate solutions and make connections and predictions. Students will analyze mathematical relationships to connect and communicate mathematical ideas. Students will display, explain, or justify mathematical ideas and arguments using precise mathematical language in written or oral communication.
3. In Algebraic Reasoning, students will build on the knowledge and skills for mathematics in Kindergarten-Grade 8 and Algebra I, continue with the development of mathematical reasoning related to algebraic understandings and processes, and deepen a foundation for studies in subsequent mathematics courses. Students will broaden their knowledge of functions and relationships, including linear, quadratic, square root, rational, cubic, cube root, exponential, absolute value, and logarithmic functions. Students will study these functions through analysis and application that includes explorations of patterns and structure, number and algebraic methods, and modeling from data using tools that build to workforce and college readiness such as probes, measurement tools, and software tools, including spreadsheets.
4. Statements that contain the word "including" reference content that must be mastered, while those containing the phrase "such as" are intended as possible illustrative examples.

(b) Knowledge and Skills Statements

(1) Mathematical process standards. The student uses mathematical processes to acquire and demonstrate mathematical understanding. The student is expected to:

(A) apply mathematics to problems arising in everyday life, society, and the workplace;

(i) apply mathematics to problems arising in everyday life

(ii) apply mathematics to problems arising in society

(iii) apply mathematics to problems arising in the workplace

(B) use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution;

(i) use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process

(ii) use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the reasonableness of the solution

(C) select tools, including real objects, manipulatives, paper and pencil, and technology as appropriate, and techniques, including mental math, estimation, and number sense as appropriate, to solve problems;

(i) select tools, including real objects as appropriate, to solve problems

(ii) select tools, including manipulatives as appropriate, to solve problems

(iii) select tools, including paper and pencil as appropriate, to solve problems

(iv) select tools, including technology as appropriate, to solve problems

(v) select techniques, including mental math as appropriate, to solve problems

(vi) select techniques including estimation as appropriate, to solve problems

(vii) select techniques, including number sense as appropriate, to solve problems

(D) communicate mathematical ideas, reasoning, and their implications using multiple representations, including symbols, diagrams, graphs, and language as appropriate;

(i) communicate mathematical ideas using multiple representations, including symbols as appropriate

(ii) communicate mathematical ideas using multiple representations, including diagrams as appropriate

(iii) communicate mathematical ideas using multiple representations, including graphs as appropriate

(iv) communicate mathematical ideas using multiple representations, including language as appropriate

(v) communicate mathematical reasoning using multiple representations, including symbols as appropriate

(vi) communicate mathematical reasoning using multiple representations, including diagrams as appropriate

(vii) communicate mathematical reasoning using multiple representations, including graphs as appropriate

(viii) communicate mathematical reasoning using multiple representations, including language as appropriate

(ix) communicate [mathematical ideas'] implications using multiple representations, including symbols as appropriate

(x) communicate [mathematical ideas'] implications using multiple representations, including diagrams as appropriate

(xi) communicate [mathematical ideas'] implications using multiple representations, including graphs as appropriate

(xii) communicate [mathematical ideas'] implications using multiple representations, including language as appropriate

(xiii) communicate [mathematical reasoning's] implications using multiple representations, including symbols as appropriate

(xiv) communicate [mathematical reasoning's] implications using multiple representations, including diagrams as appropriate

(xv) communicate [mathematical reasoning's] implications using multiple representations, including graphs as appropriate

(xvi) communicate [mathematical reasoning's] implications using multiple representations, including language as appropriate

(E) create and use representations to organize, record, and communicate mathematical ideas;

(i) create representations to organize mathematical ideas

(ii) create representations to record mathematical ideas

(iii) create representations to communicate mathematical ideas

(iv) use representations to organize mathematical ideas

(v) use representations to record mathematical ideas

(vi) use representations to communicate mathematical ideas

(F) analyze mathematical relationships to connect and communicate mathematical ideas; and

(i) analyze mathematical relationships to connect mathematical ideas

(ii) analyze mathematical relationships to communicate mathematical ideas

(G) display, explain, or justify mathematical ideas and arguments using precise mathematical language in written or oral communication.

(i) display mathematical ideas using precise mathematical language in written or oral communication

(ii) display mathematical arguments using precise mathematical language in written or oral communication

(iii) explain mathematical ideas using precise mathematical language in written or oral communication

(iv) explain mathematical arguments using precise mathematical language in written or oral communication

(v) justify mathematical ideas using precise mathematical language in written or oral communication

(vi) justify mathematical arguments using precise mathematical language in written or oral communication

(2) Patterns and structure. The student applies mathematical processes to connect finite differences or common ratios to attributes of functions. The student is expected to:

(A) determine the patterns that identify the relationship between a function and its common ratio or related finite differences as appropriate, including linear, quadratic, cubic, and exponential functions;

(i) determine the patterns that identify the relationship between a function and its common ratio or related finite differences as appropriate, including linear functions

(ii) determine the patterns that identify the relationship between a function and its common ratio or related finite differences as appropriate, including quadratic functions

(iii) determine the patterns that identify the relationship between a function and its common ratio or related finite differences as appropriate, including cubic functions

(iv) determine the patterns that identify the relationship between a function and its common ratio or related finite differences, including exponential functions

(B) classify a function as linear, quadratic, cubic, and exponential when a function is represented tabularly using finite differences or common ratios as appropriate;

(i) classify a function as linear when a function is represented tabularly using finite differences

(ii) classify a function as quadratic when a function is represented tabularly using finite differences

(iii) classify a function as cubic when a function is represented tabularly using finite differences

(iv) classify a function as exponential when a function is represented tabularly using common ratios

(C) determine the function that models a given table of related values using finite differences and its restricted domain and range; and

(i) determine the function that models a given table of related values using finite differences

(ii) determine [a function's] restricted domain

(iii) determine [a function's] restricted range

(D) determine a function that models real-world data and mathematical contexts using finite differences such as the age of a tree and its circumference, figurative numbers, average velocity, and average acceleration.

(i) determine a function that models real-world data using finite differences

(ii) determine a function that models mathematical contexts using finite differences

(3) Patterns and structure. The student applies mathematical processes to understand the connections among representations of functions and combinations of functions, including the constant function, $f(x) = x$, $f(x) = x^2$, $f(x) = \sqrt{x}$, $f(x) = {}^1/_x$, $f(x) = x^3$, $f(x) = \sqrt[3]{x}$, $f(x) = bx$, $f(x) = |x|$, and $f(x) = \log_b(x)$ where b is 10 or e; functions and their inverses; and key attributes of these functions. The student is expected to:

(A) compare and contrast the key attributes, including domain, range, maxima, minima, and intercepts, of a set of functions such as a set comprised of a linear, a quadratic, and an exponential function or a set comprised of an absolute value, a quadratic, and a square root function tabularly, graphically, and symbolically;

(i) compare and contrast the key attributes, including domain, of a set of functions, tabularly

(ii) compare and contrast the key attributes, including domain, of a set of functions, graphically

(iii) compare and contrast the key attributes, including domain, of a set of functions, symbolically

(iv) compare and contrast the key attributes, including range, of a set of functions, tabularly

(v) compare and contrast the key attributes, including range, of a set of functions, graphically

(vi) compare and contrast the key attributes, including range, of a set of functions, symbolically

(vii) compare and contrast the key attributes, including maxima, of a set of functions, tabularly

(viii) compare and contrast the key attributes, including maxima, of a set of functions, graphically

(ix) compare and contrast the key attributes, including maxima, of a set of functions, symbolically

(x) compare and contrast the key attributes, including minima, of a set of functions, tabularly

(xi) compare and contrast the key attributes, including minima, of a set of functions, graphically

(xii) compare and contrast the key attributes, including minima, of a set of functions, symbolically

(xiii) compare and contrast the key attributes, including intercepts, of a set of functions, tabularly

(xiv) compare and contrast the key attributes, including intercepts, of a set of functions, graphically

(xv) compare and contrast the key attributes, including intercepts, of a set of functions, symbolically

(B) compare and contrast the key attributes of a function and its inverse when it exists, including domain, range, maxima, minima, and intercepts, tabularly, graphically, and symbolically;

(i) compare and contrast the key attributes of a function and its inverse when it exists, including domain, tabularly

(ii) compare and contrast the key attributes of a function and its inverse when it exists, including domain, graphically

(iii) compare and contrast the key attributes of a function and its inverse when it exists, including domain, symbolically

(iv) compare and contrast the key attributes of a function and its inverse when it exists, including range, tabularly

(v) compare and contrast the key attributes of a function and its inverse when it exists, including range, graphically

(vi) compare and contrast the key attributes of a function and its inverse when it exists, including range symbolically

(vii) compare and contrast the key attributes of a function and its inverse when it exists, including maxima, tabularly

(viii) compare and contrast the key attributes of a function and its inverse when it exists, including maxima, graphically

(ix) compare and contrast the key attributes of a function and its inverse when it exists, including maxima, symbolically

(x) compare and contrast the key attributes of a function and its inverse when it exists, including minima, tabularly

(xi) compare and contrast the key attributes of a function and its inverse when it exists, including minima, graphically

(xii) compare and contrast the key attributes of a function and its inverse when it exists, including minima, symbolically

(xiii) compare and contrast the key attributes of a function and its inverse when it exists, including intercepts, tabularly

(xiv) compare and contrast the key attributes of a function and its inverse when it exists, including intercepts, graphically

(xv) compare and contrast the key attributes of a function and its inverse when it exists, including intercepts, symbolically

(C) verify that two functions are inverses of each other tabularly and graphically such as situations involving compound interest and interest rate, velocity and braking distance, and Fahrenheit-Celsius conversions;

(i) verify that two functions are inverses of each other tabularly

(ii) verify that two functions are inverses of each other graphically

(D) represent a resulting function tabularly, graphically, and symbolically when functions are combined or separated using arithmetic operations such as combining a 20% discount and a 6% sales tax on a sale to determine $h(x)$, the total sale, $f(x) = 0.8\,x$, $g(x) = 0.06(0.8x)$, and $h(x) = f(x) + g(x)$;

(i) represent a resulting function tabularly when functions, are combined or separated using arithmetic operations

(ii) represent a resulting function graphically when functions, are combined or separated using arithmetic operations

(iii) represent a resulting function symbolically when functions, are combined or separated using arithmetic operations

(E) model a situation using function notation when the output of one function is the input of a second function such as determining a function $h(x) = g(f(x)) = 1.06(0.8x)$ for the final purchase price, $h(x)$ of an item with price x dollars representing a 20% discount, $f(x) = 0.8x$ followed by a 6% sales tax, $g(x) = 1.06x$; and

(i) Model a situation using function notation when the output of one function is the input of a second function

(F) compare and contrast a function and possible functions that can be used to build it tabularly, graphically, and symbolically such as a quadratic function that results from multiplying two linear functions.

(i) compare and contrast a function and possible functions that can be used to build it tabularly

(ii) compare and contrast a function and possible functions that can be used to build it graphically

(iii) compare and contrast a function and possible functions that can be used to build it symbolically

(4) Number and algebraic methods. The student applies mathematical processes to simplify and perform operations on functions represented in a variety of ways, including real-world situations. The student is expected to:

(A) connect tabular representations to symbolic representations when adding, subtracting, and multiplying polynomial functions arising from mathematical and real-world situations such as applications involving surface area and volume;

(i) connect tabular representations to symbolic representations when adding polynomial functions arising from mathematical situations

(ii) connect tabular representations to symbolic representations when adding polynomial functions arising from real-world situations

(iii) connect tabular representations to symbolic representations when subtracting polynomial functions arising from mathematical situations

(iv) connect tabular representations to symbolic representations when subtracting polynomial functions arising from real-world situations

(v) connect tabular representations to symbolic representations when multiplying polynomial functions arising from mathematical situations

(vi) connect tabular representations to symbolic representations when multiplying polynomial functions arising from real-world situations

(B) compare and contrast the results when adding two linear functions and multiplying two linear functions that are represented tabularly, graphically, and symbolically;

(i) compare and contrast the results when adding two linear functions that are represented tabularly

(ii) compare and contrast the results when adding two linear functions that are represented graphically

(iii) compare and contrast the results when adding two linear functions that are represented symbolically

(iv) compare and contrast the results when multiplying two linear functions that are represented tabularly

(v) compare and contrast the results when multiplying two linear functions that are represented graphically

(vi) compare and contrast the results when multiplying two linear functions that are represented symbolically

(C) determine the quotient of a polynomial function of degree three and of degree four when divided by a polynomial function of degree one and of degree two when represented tabularly and symbolically; and

(i) determine the quotient of a polynomial function of degree three when divided by a polynomial function of degree one when represented tabularly

(ii) determine the quotient of a polynomial function of degree three when divided by a polynomial function of degree one when represented symbolically

(iii) determine the quotient of a polynomial function of degree three when divided by a polynomial function of degree two when represented tabularly

(iv) determine the quotient of a polynomial function of degree three when divided by a polynomial function of degree two when represented symbolically

(v) determine the quotient of a polynomial function of degree four when divided by a polynomial function of degree one when represented tabularly

(vi) determine the quotient of a polynomial function of degree four when divided by a polynomial function of degree one when represented symbolically

(vii) determine the quotient of a polynomial function of degree four when divided by a polynomial function of degree two when represented tabularly

(viii) determine the quotient of a polynomial function of degree four when divided by a polynomial function of degree two when represented symbolically

(D) determine the linear factors of a polynomial function of degree two and of degree three when represented symbolically and tabularly and graphically where appropriate.

(i) determine the linear factors of a polynomial function of degree two when represented symbolically

(ii) determine the linear factors of a polynomial function of degree two when represented tabularly (where appropriate)

(iii) determine the linear factors of a polynomial function of degree two when represented graphically (where appropriate)

(iv) determine the linear factors of a polynomial function of degree three when represented symbolically

(v) determine the linear factors of a polynomial function of degree three when represented tabularly (where appropriate)

(vi) determine the linear factors of a polynomial function of degree three when represented graphically (where appropriate)

(5) Number and algebraic methods. The student applies mathematical processes to represent, simplify, and perform operations on matrices and to solve systems of equations using matrices. The student is expected to:

(A) add and subtract matrices;

(i) add matrices

(ii) subtract matrices

(B) multiply matrices;

(i) multiply matrices

(C) multiply matrices by a scalar;

(i) multiply matrices by a scalar

(D) represent and solve systems of two linear equations arising from mathematical and real-world situations using matrices; and

(i) represent systems of two linear equations arising from mathematical situations using matrices

(ii) represent systems of two linear equations arising from real-world situations using matrices

(iii) solve systems of two linear equations arising from mathematical situations using matrices

(iv) solve systems of two linear equations arising from real-world situations using matrices

(E) represent and solve systems of three linear equations arising from mathematical and real-world situations using matrices and technology.

(i) represent systems of three linear equations arising from mathematical situations using matrices

(ii) represent systems of three linear equations arising from mathematical situations using technology

(iii) represent systems of three linear equations arising from real-world situations using matrices

(iv) represent systems of three linear equations arising from real-world situations using technology

(v) solve systems of three linear equations arising from mathematical situations using matrices

(vi) solve systems of three linear equations arising from mathematical situations using technology

(vii) solve systems of three linear equations arising from real-world situations using matrices

(viii) solve systems of three linear equations arising from real-world situations using technology

(6) Number and algebraic methods. The student applies mathematical processes to estimate and determine solutions to equations resulting from functions and real-world applications with fluency. The student is expected to:

(A) estimate a reasonable input value that results in a given output value for a given function, including quadratic, rational, and exponential functions;

(i) estimate a reasonable input value that results in a given output value for a given function, including quadratic functions

(ii) estimate a reasonable input value that results in a given output value for a given function, including rational functions

(iii) estimate a reasonable input value that results in a given output value for a given function, including exponential functions

(B) solve equations arising from questions asked about functions that model real-world applications, including linear and quadratic functions, tabularly, graphically, and symbolically; and

(i) solve equations arising from questions asked about functions that model real-world applications, including linear functions, tabularly

(ii) solve equations arising from questions asked about functions that model real-world applications, including linear functions, graphically

(iii) solve equations arising from questions asked about functions that model real-world applications, including linear functions, symbolically

(iv) solve equations arising from questions asked about functions that model real-world applications, including quadratic functions, tabularly

(v) solve equations arising from questions asked about functions that model real-world applications, including quadratic functions, graphically

(vi) solve equations arising from questions asked about functions that model real-world applications, including quadratic functions, symbolically

(C) approximate solutions to equations arising from questions asked about exponential, logarithmic, square root, and cubic functions that model real-world applications tabularly and graphically.

(i) approximate solutions to equations arising from questions asked about exponential functions that model real-world applications tabularly

(ii) approximate solutions to equations arising from questions asked about exponential functions that model real-world applications graphically

(iii) approximate solutions to equations arising from questions asked about logarithmic functions that model real-world applications tabularly

(iv) approximate solutions to equations arising from questions asked about logarithmic functions that model real-world applications graphically

(v) approximate solutions to equations arising from questions asked about square root functions that model real-world applications tabularly

(vi) approximate solutions to equations arising from questions asked about square root functions that model real-world applications graphically

(vii) approximate solutions to equations arising from questions asked about cubic functions that model real-world applications tabularly

(viii) approximate solutions to equations arising from questions asked about cubic functions that model real-world applications graphically

(7) Modeling from data. The student applies mathematical processes to analyze and model data based on real-world situations with corresponding functions. The student is expected to:

(A) represent domain and range of a function using interval notation, inequalities, and set (builder) notation;

(i) represent the domain of a function using interval notation

(ii) represent the domain of a function using inequalities

(iii) represent the domain of a function using set (builder) notation

(iv) represent the range of a function using interval notation

(v) represent the range of a function using inequalities

(vi) represent the range of a function using set (builder) notation

(B) compare and contrast between the mathematical and reasonable domain and range of functions modeling real-world situations, including linear, quadratic, exponential, and rational functions;

(i) compare and contrast between the mathematical and reasonable domain of functions modeling real-world situations, including linear functions

(ii) compare and contrast between the mathematical and reasonable domain of functions modeling real-world situations, including quadratic functions

(iii) compare and contrast between the mathematical and reasonable domain of functions modeling real-world situations, including exponential functions

(iv) compare and contrast between the mathematical and reasonable domain of functions modeling real-world situations, including rational functions

(v) compare and contrast between the mathematical and reasonable range of functions modeling real-world situations, including linear functions

(vi) compare and contrast between the mathematical and reasonable range of functions modeling real-world situations, including quadratic functions

(vii) compare and contrast between the mathematical and reasonable range of functions modeling real-world situations, including exponential functions

(viii) compare and contrast between the mathematical and reasonable range of functions modeling real-world situations, including rational functions

(C) determine the accuracy of a prediction from a function that models a set of data compared to the actual data using comparisons between average rates of change and finite differences such as gathering data from an emptying tank and comparing the average rate of change of the volume or the second differences in the volume to key attributes of the given model;

(i) determine the accuracy of a prediction from a function that models a set of data compared to the actual data using comparisons between average rates of change and finite differences

(D) determine an appropriate function model, including linear, quadratic, and exponential functions, for a set of data arising from real-world situations using finite differences and average rates of change; and

(i) determine an appropriate function model, including linear functions, for a set of data arising from real-world situations using finite differences

(ii) determine an appropriate function model, including linear functions, for a set of data arising from real-world situations using average rates of change

(iii) determine an appropriate function model, including quadratic functions, for a set of data arising from real-world situations using finite differences

(iv) determine an appropriate function model, including quadratic functions, for a set of data arising from real-world situations using average rates of change

(v) determine an appropriate function model, including exponential functions, for a set of data arising from real-world situations using finite differences

(vi) determine an appropriate function model, including exponential functions, for a set of data arising from real-world situations using average rates of change

(E) determine if a given linear function is a reasonable model for a set of data arising from a real-world situation

(i) determine if a given linear function is a reasonable model for a set of data arising from a real-world situation