



TEKS Vertical Alignment for STAAR Alternate

Mathematics

Pre-kindergarten through End-of-Course

Number, Operation, and Quantitative Reasoning

Counting skills. The student shows basic counting readiness and counting by using nonverbal and verbal means (Pre-K.V.A).

Number, operation, and quantitative reasoning. The student uses numbers to name quantities (K.1). The student describes order of events or objects (K.2). The student uses whole numbers to describe and compare quantities (1.1). The student understands how place value is used to represent whole numbers (2.1). The student adds and subtracts whole numbers to solve problems (2.3). The student uses place value to communicate about increasingly large whole numbers in verbal and written form, including money (3.1). The student uses place value to represent whole numbers and decimals (4.1; 5.1). The student represents and uses rational numbers in a variety of equivalent forms (6.1). The student represents and uses numbers in a variety of equivalent forms (7.1). The student understands that different forms of numbers are appropriate for different situations (8.1). The student is expected to

Place Value and Equivalent Forms of Numbers

- know that objects, or parts of an object, can be counted (Pre-K)
- use words to rote count from 1 to 30 (Pre-K)
- count 1-10 items, with one count per item (Pre-K)
- demonstrate that the order of the counting sequence is always the same, regardless of what is counted (Pre-K)
- count up to 10 items, and demonstrate that the last count indicates how many items were counted (Pre-K)
- demonstrate understanding that when counting, the items can be chosen in any order (Pre-K)
- use the verbal ordinal terms (Pre-K)
- verbally identify, without counting, the number of objects from 1 to 5 (Pre-K)
- recognize one-digit numerals, 0-9 (Pre-K)
- use one-to-one correspondence and language such as more than, same number as, or two less than to describe relative sizes of sets of concrete objects (K)
- use sets of concrete objects to represent quantities given in verbal or written form (through 20) (K)
- use numbers to describe how many objects are in a set (through 20) using verbal and symbolic descriptions (K)
- use language such as before or after to describe relative position in a sequence of events or objects (K)
- name the ordinal positions in a sequence such as first, second, third, etc. (K)
- compare and order whole numbers up to 99 (less than, greater than, or equal to) using sets of concrete objects and pictorial models (1)
- create sets of tens and ones using concrete objects to describe, compare, and order whole numbers (1)
- identify individual coins by name and value and describe relationships among them (1)
- read and write numbers to 99 to describe sets of concrete objects (1)
- use concrete models of hundreds, tens, and ones to represent a given whole number (up to 999) in various ways (2)
- use place value to read, write, and describe the value of whole numbers to 999 (2)
- use place value to compare and order whole numbers to 999 and record the comparisons using numbers and symbols ($<$, $=$, $>$) (2)
- determine the value of a collection of coins up to one dollar (2)

- describe how the cent symbol, dollar symbol, and the decimal point are used to name the value of a collection of coins (2)
- use place value to read, write (in symbols and words), and describe the value of whole numbers through 999,999 (3)
- use place value to compare and order whole numbers through 9,999 (3)
- determine the value of a collection of coins and bills (3)
- use place value to read, write, compare, and order whole numbers through 999,999,999 (4)
- use place value to read, write, compare, and order decimals involving tenths and hundredths, including money, using concrete objects and pictorial models (4)
- use place value to read, write, compare, and order whole numbers through 999,999,999,999 (5)
- use place value to read, write, compare, and order decimals through the thousandths place (5)
- compare and order non-negative rational numbers (6)
- generate equivalent forms of rational numbers including whole numbers, fractions, and decimals (6)
- use integers to represent real-life situations (6)
- write prime factorizations using exponents (6)
- identify factors of a positive integer, common factors, and the greatest common factor of a set of positive integers (6)
- identify multiples of a positive integer and common multiples and the least common multiple of a set of positive integers (6)
- compare and order integers and positive rational numbers (7)
- convert between fractions, decimals, whole numbers, and percents mentally, on paper, or with a calculator (7)
- represent squares and square roots using geometric models (7)
- compare and order rational numbers in various forms including integers, percents, and positive and negative fractions and decimals (8)
- select and use appropriate forms of rational numbers to solve real-life problems including those involving proportional relationships (8)
- approximate (mentally and with calculators) the value of irrational numbers as they arise from problem situations (such as π , $\sqrt{2}$) (8)
- express numbers in scientific notation, including negative exponents, in appropriate problem situations (8)
- compare and order real numbers with a calculator (8)

Number, operation, and quantitative reasoning. The student recognizes that there are quantities less than a whole (K.3). The student uses pairs of whole numbers to describe fractional parts of whole objects or sets of objects (1.2). The student describes how fractions are used to name parts of whole objects or sets of objects (2.2). The student uses fraction names and symbols (with denominators of 12 or less) to describe fractional parts of whole objects or sets of objects (3.2). The student describes and compares fractional parts of whole objects or sets of objects (4.2). The student uses fractions in problem-solving situations (5.2). The student is expected to

Fractions

- share a whole by separating it into two equal parts (K)
- explain why a given part is half of the whole (K)
- separate a whole into two, three, or four equal parts and use appropriate language to describe the parts such as three out of four equal parts (1)

- use appropriate language to describe part of a set such as three out of the eight crayons are red (1)
- use concrete models to represent and name fractional parts of a whole object (with denominators of 12 or less) (2)
- use concrete models to represent and name fractional parts of a set of objects (with denominators of 12 or less) (2)
- use concrete models to determine if a fractional part of a whole is closer to 0, $\frac{1}{2}$, or 1 (2)
- construct concrete models of fractions (3)
- compare fractional parts of whole objects or sets of objects in a problem situation using concrete models (3)
- use fraction names and symbols to describe fractional parts of whole objects or sets of objects (3)
- construct concrete models of equivalent fractions for fractional parts of whole objects (3)
- use concrete objects and pictorial models to generate equivalent fractions (4)
- model fraction quantities greater than one using concrete objects and pictorial models (4)
- compare and order fractions using concrete objects and pictorial models (4)
- relate decimals to fractions that name tenths and hundredths using concrete objects and pictorial models (4)
- generate a fraction equivalent to a given fraction such as $\frac{1}{2}$ and $\frac{3}{6}$ or $\frac{4}{12}$ and $\frac{1}{3}$ (5)
- generate a mixed number equivalent to a given improper fraction or generate an improper fraction equivalent to a given mixed number (5)
- compare two fractional quantities in problem-solving situations using a variety of methods, including common denominators (5)
- use models to relate decimals to fractions that name tenths, hundredths, and thousandths (5)

Adding to/taking away skills. The student uses informal and formal strategies to make a collection larger or smaller (Pre-K.V.B).

Number, operation, and quantitative reasoning. The student models addition (joining) and subtraction (separating) (K.4). The student recognizes and solves problems in addition and subtraction situations (1.3). The student adds and subtracts whole numbers to solve problems (2.3). The student models multiplication and division (2.4). The student adds and subtracts to solve meaningful problems involving whole numbers (3.3). The student recognizes and solves problems in multiplication and division situations (3.4). The student estimates to determine reasonable results (3.5; 4.5; 5.4). The student adds and subtracts to solve meaningful problems involving whole numbers and decimals (4.3). The student multiplies and divides to solve meaningful problems involving whole numbers (4.4). The student adds, subtracts, multiplies, and divides to solve meaningful problems (5.3). The student adds, subtracts, multiplies, and divides to solve problems and justify solutions (6.2). The student adds, subtracts, multiplies, or divides to solve problems and justify solutions (7.2). The student selects and uses appropriate operations to solve problems and justify solutions (8.2).

Foundations for functions. The student understands the importance of the skills required to manipulate symbols in order to solve problems and uses the necessary algebraic skills required to simplify algebraic expressions and solve equations and inequalities in problem situations (A.4). The student is expected to

Operations and Reasoning: Addition and Subtraction

- use concrete models or make a verbal word problem for adding up to 5 objects (Pre-K)

- use concrete models or make a verbal word problem for subtracting 1–5 objects from a set (Pre-K)
- model and create addition and subtraction problems in real situations with concrete objects (K)
- model and create addition and subtraction problem situations with concrete objects and write corresponding number sentences (1)
- use concrete and pictorial models to apply basic addition and subtraction facts (up to $9 + 9 = 18$ and $18 - 9 = 9$) (1)
- recall and apply basic addition and subtraction facts (to 18) (2)
- model addition and subtraction of two-digit numbers with objects, pictures, words, and numbers (2)
- select addition or subtraction to solve problems using two-digit numbers, whether or not regrouping is necessary (2)
- model addition and subtraction using pictures, words, and numbers (3)
- select addition or subtraction and use the operation to solve problems involving whole numbers through 999 (3)
- use addition and subtraction to solve problems involving whole numbers (4)
- add and subtract decimals to the hundredths place using concrete objects and pictorial models (4)
- use addition and subtraction to solve problems involving whole numbers and decimals (5)
- model situations using addition and/or subtraction involving fractions with like denominators using concrete objects, pictures, words, and numbers (5)
- model addition and subtraction situations involving fractions with objects, pictures, words, and numbers (6)
- use addition and subtraction to solve problems involving fractions and decimals (6)

Operations and Reasoning: Multiplication and Division

- use informal strategies to share or divide up to 10 items equally (Pre-K)
- model, create, and describe multiplication situations in which equivalent sets of concrete objects are joined (2)
- model, create, and describe division situations in which a set of concrete objects is separated into equivalent sets (2)
- learn and apply multiplication facts through 12 by 12 using concrete models and objects (3)
- solve and record multiplication problems (up to two digits times one digit) (3)
- use models to solve division problems and use number sentences to record the solutions (3)
- model factors and products using arrays and area models (4)
- represent multiplication and division situations in picture, word, and number form (4)
- recall and apply multiplication facts through 12×12 (4)
- use multiplication to solve problems (no more than two digits times two digits without technology) (4)
- use division to solve problems (no more than one-digit divisors and three-digit dividends without technology) (4)
- use multiplication to solve problems involving whole numbers (no more than three digits times two digits without technology) (5)
- use division to solve problems involving whole numbers (no more than two-digit divisors and three-digit dividends without technology), including interpreting the remainder within a given context (5)
- identify common factors of a set of whole numbers (5)

- use multiplication and division of whole numbers to solve problems including situations involving equivalent ratios and rates (6)
- use order of operations to simplify whole number expressions (without exponents) in problem solving situations (6)
- represent multiplication and division situations involving fractions and decimals with models, including concrete objects, pictures, words, and numbers (7)
- use addition, subtraction, multiplication, and division to solve problems involving fractions and decimals (7)
- use models, such as concrete objects, pictorial models, and number lines, to add, subtract, multiply, and divide integers and connect the actions to algorithms (7)
- use division to find unit rates and ratios in proportional relationships such as speed, density, price, recipes, and student-teacher ratio (7)
- simplify numerical expressions involving order of operations and exponents (7)
- select and use appropriate operations to solve problems and justify the selections (7)
- select appropriate operations to solve problems involving rational numbers and justify the selections (8)
- use appropriate operations to solve problems involving rational numbers in problem situations (8)
- use multiplication by a given constant factor (including unit rate) to represent and solve problems involving proportional relationships including conversions between measurement systems (8)
- find specific function values, simplify polynomial expressions, transform and solve equations, and factor as necessary in problem situations (Alg)
- use the commutative, associative, and distributive properties to simplify algebraic expressions (Alg)
- connect equation notation with function notation, such as $y = x + 1$ and $f(x) = x + 1$ (Alg)

Operations and Reasoning: Estimation and Reasonableness

- round whole numbers to the nearest ten or hundred to approximate reasonable results in problem situations (3)
- use strategies including rounding and compatible numbers to estimate solutions to addition and subtraction problems (3)
- round whole numbers to the nearest ten, hundred, or thousand to approximate reasonable results in problem situations (4)
- use strategies including rounding and compatible numbers to estimate solutions to multiplication and division problems (4)
- use strategies, including rounding and compatible numbers to estimate solutions to addition, subtraction, multiplication, and division problems (5)
- estimate and round to approximate reasonable results and to solve problems where exact answers are not required (6)
- determine the reasonableness of a solution to a problem (7)
- evaluate a solution for reasonableness (8)

Patterns, Relationships, and Algebraic Thinking

Classification and patterns skills. The student sorts and classifies objects using one or more attributes and uses attributes of objects to duplicate and create patterns (Pre-K.V.E).

Patterns, relationships, and algebraic thinking. The student identifies, extends, and creates patterns (K.5). The student uses patterns to make predictions (K.6). The student uses repeating patterns and additive patterns to make predictions (1.4). The student recognizes patterns in numbers and operations (1.5). The student uses patterns in numbers and operations (2.5). The student uses patterns to describe relationships and make predictions (2.6). The student uses patterns to solve problems (3.6). The student uses patterns in multiplication and division (4.6). The student is expected to

Patterns

- recognize and create patterns (Pre-K)
- identify, extend, and create patterns of sounds, physical movement, and concrete objects (K)
- use patterns to predict what comes next, including cause-and-effect relationships (K)
- count by ones to 100 (K)
- identify, describe, and extend concrete and pictorial patterns in order to make predictions and solve problems (1)
- use patterns to skip count by twos, fives, and tens (1)
- find patterns in numbers, including odd and even (1)
- compare and order whole numbers using place value (1)
- use patterns to develop strategies to solve basic addition and basic subtraction problems (1)
- identify patterns in related addition and subtraction sentences (fact families for sums to 18) such as $2 + 3 = 5$, $3 + 2 = 5$, $5 - 2 = 3$, and $5 - 3 = 2$ (1)
- find patterns in numbers such as in a 100s chart (2)
- use patterns in place value to compare and order whole numbers through 999 (2)
- use patterns and relationships to develop strategies to remember basic addition and subtraction facts. Determine patterns in related addition and subtraction number sentences (including fact families) such as $8 + 9 = 17$, $9 + 8 = 17$, $17 - 8 = 9$, and $17 - 9 = 8$ (2)
- generate a list of paired numbers based on a real-life situation such as number of tricycles related to number of wheels (2)
- identify patterns in a list of related number pairs based on a real-life situation and extend the list (2)
- identify, describe, and extend repeating and additive patterns to make predictions and solve problems (2)
- identify and extend whole-number and geometric patterns to make predictions and solve problems (3)
- identify patterns in multiplication facts using concrete objects, pictorial models, or technology (3)
- identify patterns in related multiplication and division sentences (fact families) such as $2 \times 3 = 6$, $3 \times 2 = 6$, $6 \div 2 = 3$, $6 \div 3 = 2$ (3)
- use patterns and relationships to develop strategies to remember basic multiplication and division facts (such as the patterns in related multiplication and division number sentences (fact families) such as $9 \times 9 = 81$ and $81 \div 9 = 9$) (4)
- use patterns to multiply by 10 and 100 (4)

Patterns, relationships, and algebraic thinking. The student solves problems involving direct proportional relationships (6.3; 7.3). The student represents a relationship in numerical, geometric, verbal, and symbolic form (7.4). The student identifies proportional or non-proportional linear relationships in problem situations and solves problems (8.3).

Foundations for functions. The student understands that a function represents a dependence of one quantity on another and can be described in a variety of ways (A.1). The student understands how algebra can be used to express generalizations and recognizes and uses the power of symbols to represent situations (A.3).

Linear functions. The student understands the meaning of the slope and intercepts of the graphs of linear functions and zeros of linear functions and interprets and describes the effects of changes in parameters of linear functions in real-world and mathematical situations (A.6).

Quadratic and other nonlinear functions. The student understands that the graphs of quadratic functions are affected by the parameters of the function and can interpret and describe the effects of changes in the parameters of quadratic functions (A.9). The student is expected to

Relationships

- use ratios to describe proportional situations (6)
- represent ratios and percents with concrete models, fractions, and decimals (6)
- use ratios to make predictions in proportional situations (6)
- estimate and find solutions to application problems involving percent (7)
- estimate and find solutions to application problems involving proportional relationships such as similarity, scaling, unit costs, and related measurement units (7)
- use words and symbols to describe the relationship between the terms in an arithmetic sequence (with a constant rate of change) and their positions in the sequence (7)
- compare and contrast proportional and non-proportional linear relationships (8)
- estimate and find solutions to application problems involving percents and other proportional relationships such as similarity and rates (8)
- describe independent and dependent quantities in functional relationships (Alg)
- gather and record data and use data sets to determine functional relationships between quantities (Alg)
- describe functional relationships for given problem situations and write equations or inequalities to answer questions arising from the situations (Alg)
- represent relationships among quantities using concrete models, tables, graphs, diagrams, verbal descriptions, equations, and inequalities (Alg)
- interpret and make decisions, predictions, and critical judgments from functional relationships (Alg)
- use symbols to represent unknowns and variables (Alg)
- look for patterns and represent generalizations algebraically (Alg)
- develop the concept of slope as rate of change and determine slopes from graphs, tables, and algebraic representations (Alg)
- interpret the meaning of slope and intercepts in situations using data, symbolic representations, or graphs (Alg)
- investigate, describe, and predict the effects of changes in m and b on the graph of $y = mx + b$ (Alg)
- graph and write equations of lines given characteristics such as two points, a point and a slope, or a slope and y -intercept (Alg)
- determine the intercepts of the graphs of linear functions and zeros of linear functions from graphs, tables, and algebraic representations (Alg)

- interpret and predict the effects of changing slope and y -intercept in applied situations (Alg)
- relate direct variation to linear functions and solve problems involving proportional change (Alg)
- determine the domain and range for quadratic functions in given situations (Alg)
- investigate, describe, and predict the effects of changes in a on the graph of $y = ax^2 + c$ (Alg)
- investigate, describe, and predict the effects of changes in c on the graph of $y = ax^2 + c$ (Alg)
- analyze graphs of quadratic functions and draw conclusions (Alg)

Patterns, relationships, and algebraic thinking. The student uses lists, tables, and charts to express patterns and relationships (3.7). The student uses organizational structures to analyze and describe patterns and relationships (4.7). The student makes generalizations based on observed patterns and relationships (5.5). The student describes relationships mathematically (5.6). The student uses letters as variables in mathematical expressions to describe how one quantity changes when a related quantity changes (6.4). The student uses letters to represent an unknown in an equation (6.5). The student represents a relationship in numerical, geometric, verbal, and symbolic form (7.4). The student uses equations to solve problems (7.5). The student makes connections among various representations of a numerical relationship (8.4). The student uses graphs, tables, and algebraic representations to make predictions and solve problems (8.5).

Foundations for functions. The student uses the properties and attributes of functions (A.2).

Linear functions. The student understands that linear functions can be represented in different ways and translates among their various representations (A.5). The student formulates equations and inequalities based on linear functions, uses a variety of methods to solve them, and analyzes the solutions in terms of the situation (A.7). The student formulates systems of linear equations from problem situations, uses a variety of methods to solve them, and analyzes the solutions in terms of the situation (A.8).

Quadratic and other nonlinear functions. The student understands there is more than one way to solve a quadratic equation and solves them using appropriate methods (A.10). The student understands there are situations modeled by functions that are neither linear nor quadratic and models the situations (A.11). The student is expected to

Graphical Representations

- generate a table of paired numbers based on a real-life situation such as insects and legs (3)
- identify and describe patterns in a table of related number pairs based on a meaningful problem and extend the table (3)
- describe the relationship between two sets of related data such as ordered pairs in a table (4)
- describe the relationship between sets of data in graphic organizers such as lists, tables, charts, and diagrams (5)
- identify prime and composite numbers using concrete objects, pictorial models, and patterns in factor pairs (5)
- select from and use diagrams and equations such as $y = 5 + 3$ to represent meaningful problem situations (5)
- use tables and symbols to represent and describe proportional and other relationships such as those involving conversions, arithmetic sequences (with a constant rate of change), perimeter and area (6)

- use tables of data to generate formulas representing relationships involving perimeter, area, volume of a rectangular prism, etc. (6)
- formulate equations from problem situations described by linear relationships (6)
- generate formulas involving unit conversions within the same system (customary and metric), perimeter, area, circumference, volume, and scaling (7)
- graph data to demonstrate relationships in familiar concepts such as conversions, perimeter, area, circumference, volume, and scaling (7)
- use concrete and pictorial models to solve equations and use symbols to record the actions (7)
- formulate problem situations when given a simple equation and formulate an equation when given a problem situation (7)
- generate a different representation of data given another representation of data (such as a table, graph, equation, or verbal description) (8)
- predict, find, and justify solutions to application problems using appropriate tables, graphs, and algebraic equations (8)
- find and evaluate an algebraic expression to determine any term in an arithmetic sequence (with a constant rate of change) (8)
- identify and sketch the general forms of linear ($y = x$) and quadratic ($y = x^2$) parent functions (Alg)
- identify mathematical domains and ranges and determine reasonable domain and range values for given situations, both continuous and discrete (Alg)
- interpret situations in terms of given graphs or creates situations that fit given graphs (Alg)
- collect and organize data, make and interpret scatterplots (including recognizing positive, negative, or no correlation for data approximating linear situations), and model, predict, and make decisions and critical judgments in problem situations (Alg)
- determine whether or not given situations can be represented by linear functions (Alg)
- determine the domain and range for linear functions in given situations (Alg)
- use, translate, and make connections among algebraic, tabular, graphical, or verbal descriptions of linear functions (Alg)
- analyze situations involving linear functions and formulate linear equations or inequalities to solve problems (Alg)
- investigate methods for solving linear equations and inequalities using concrete models, graphs, and the properties of equality, select a method, and solve the equations and inequalities (Alg)
- interpret and determine the reasonableness of solutions to linear equations and inequalities (Alg)
- analyze situations and formulate systems of linear equations in two unknowns to solve problems (Alg)
- solve systems of linear equations using concrete models, graphs, tables, and algebraic methods (Alg)
- interpret and determine the reasonableness of solutions to systems of linear equations (Alg)
- solve quadratic equations using concrete models, tables, graphs, and algebraic methods (Alg)
- make connections among the solutions (roots) of quadratic equations, the zeros of their related functions, and the horizontal intercepts (x -intercepts) of the graph of the function (Alg)
- use patterns to generate the laws of exponents and apply them in problem-solving situations (Alg)
- analyze data and represent situations involving inverse variation using concrete models, tables, graphs or algebraic methods (Alg)

- analyze data and represent situations involving exponential growth and decay using concrete models, tables, graphs, or algebraic methods (Alg)

Geometry and Spatial Reasoning

Geometry and spatial sense skills. The student recognizes, describes, and names attributes of shapes (Pre-K.V.C).

Geometry and spatial reasoning. The student uses attributes to determine how objects are alike and different (K.8). The student recognizes attributes of two- and three-dimensional geometric figures (K.9). The student uses attributes to identify two- and three-dimensional geometric figures. The student compares and contrasts two- and three-dimensional geometric figures or both (1.6; 2.7). The student uses formal geometric vocabulary (3.8). The student identifies and describes attributes of geometric figures using formal geometric language (4.8). The student generates geometric definitions using critical attributes (5.7). The student uses geometric vocabulary to describe angles, polygons, and circles (6.6). The student compares and classifies two- and three-dimensional figures using geometric vocabulary and properties (7.6).

Geometric structure. The student analyzes geometric relationships in order to make and verify conjectures (G.2). The student applies logical reasoning to justify and prove mathematical statements (G.3). The student uses a variety of representations to describe geometric relationships and solve problems (G.4).

Congruence and the geometry of size. The student analyzes properties and describes relationships in geometric figures (G.9). The student is expected to

Attributes of Geometric Figures

- name common shapes (Pre-K)
- create shapes (Pre-K)
- describe and identify an object by its attributes using informal language (K)
- compare two objects based on their attributes (K)
- sort a variety of objects including two- and three-dimensional geometric figures according to their attributes and describe how the objects are sorted (K)
- describe and compare the attributes of real-life objects such as balls, boxes, cans, and cones or models of three-dimensional geometric figures (K)
- recognize shapes in real-life three-dimensional geometric figures or models of three-dimensional geometric figures (K)
- describe, identify, and compare circles, triangles, rectangles, and squares (a special type of rectangle) (K)
- describe and identify two-dimensional geometric figures, including circles, triangles, rectangles, and squares (a special type of rectangle) (1)
- describe and identify three-dimensional geometric figures, including spheres, rectangular prisms (including cubes), cylinders, and cones (1)
- describe and identify two- and three-dimensional geometric figures in order to sort them according to a given attribute using informal and formal language (1)
- use concrete models to combine two-dimensional geometric figures to make new geometric figures (1)

- describe attributes (the number of vertices, faces, edges, sides) of two- and three-dimensional geometric figures such as circles, polygons, spheres, cones, cylinders, prisms, and pyramids, etc. (2)
- use attributes to describe how 2 two-dimensional figures or 2 three-dimensional geometric figures are alike or different (2)
- cut two-dimensional geometric figures apart and identify the new geometric figures formed (2)
- identify, classify, and describe two- and three-dimensional geometric figures by their attributes (3)
- compare two-dimensional figures, three-dimensional figures, or both by their attributes using formal geometry vocabulary (3)
- identify and describe right, acute, and obtuse angles (4)
- identify and describe parallel and intersecting (including perpendicular) lines using concrete objects and pictorial models (4)
- use essential attributes to define two- and three-dimensional geometric figures (4)
- identify essential attributes including parallel, perpendicular, and congruent parts of two- and three-dimensional geometric figures (5)
- use angle measurements to classify angles as acute, obtuse, or right (6)
- identify relationships involving angles in triangles and quadrilaterals (6)
- describe the relationship between radius, diameter, and circumference of a circle (6)
- use angle measurements to classify pairs of angles as complementary or supplementary (7)
- use properties to classify triangles and quadrilaterals (7)
- use properties to classify three-dimensional figures, including pyramids, cones, prisms, and cylinders (7)
- use critical attributes to define similarity (7)
- use constructions to explore attributes of geometric figures and to make conjectures about geometric relationships (Geom)

Geometric Reasoning

- make conjectures about angles, lines, polygons, circles, and three-dimensional figures and determine the validity of the conjectures, choosing from a variety of approaches such as coordinate, transformational, or axiomatic (Geom)
- determine the validity of a conditional statement, its converse, inverse, and contrapositive (Geom)
- construct and justify statements about geometric figures and their properties (Geom)
- use logical reasoning to prove statements are true and find counter examples to disprove statements that are false (Geom)
- use inductive reasoning to formulate a conjecture (Geom)
- use deductive reasoning to prove a statement (Geom)
- select an appropriate representation (concrete, pictorial, graphical, verbal, or symbolic) in order to solve problems (Geom)
- formulate and test conjectures about the properties of parallel and perpendicular lines based on explorations and concrete models (Geom)
- formulate and test conjectures about the properties and attributes of polygons and their component parts based on explorations and concrete models (Geom)
- formulate and test conjectures about the properties and attributes of circles and the lines that intersect them based on explorations and concrete models (Geom)

- analyze the characteristics of polyhedra and other three-dimensional figures and their component parts based on explorations and concrete models (Geom)

Geometry and spatial sense skills. The student recognizes, describes, and names attributes of shapes (Pre-K.V.C).

Geometry and spatial reasoning. The student describes the relative positions of objects (K.7). The student recognizes congruence and symmetry (3.9). The student connects transformations to congruence and symmetry (4.9). The student models transformations (5.8).

Congruence and the geometry of size. The student applies the concept of congruence to justify properties of figures and solve problems (G.10). The student is expected to

Symmetry and Transformations

- demonstrate use of location words (such as “over,” “under,” “above,” “on,” “beside,” “next to,” “between,” “in front of,” “near,” “far,” etc.) (Pre-K)
- slide, flip, and turn shapes to demonstrate that the shapes remain the same (Pre-K)
- describe one object in relation to another using informal language such as over, under, above, and below (K)
- place an object in a specified position (K)
- identify congruent two-dimensional figures (3)
- create two-dimensional figures with lines of symmetry using concrete models and technology (3)
- identify lines of symmetry in two-dimensional geometric figures (3)
- demonstrate translations, reflections, and rotations using concrete models (4)
- use translations, reflections, and rotations to verify that two shapes are congruent (4)
- use reflections to verify that a shape has symmetry (4)
- sketch the results of translations, rotations, and reflections on a Quadrant I coordinate grid (5)
- identify the transformation that generates one figure from the other when given two congruent figures on a Quadrant I coordinate grid (5)
- use congruence transformations to make conjectures and justify properties of geometric figures including figures represented on a coordinate plane (Geom)
- justify and apply triangle congruence relationships (Geom)

Geometry and spatial reasoning. The student recognizes that a line can be used to represent a set of numbers and its properties (2.8). The student recognizes that a line can be used to represent numbers and fractions and their properties and relationships (3.10). The student recognizes the connection between numbers and their properties and points on a line (4.10). The student recognizes the connection between ordered pairs of numbers and locations of points on a plane (5.9). The student uses coordinate geometry to identify location in two dimensions (6.7). The student uses coordinate geometry to describe location on a plane (7.7). The student uses geometry to model and describe the physical world (8.7).

Dimensionality and the geometry of location. The student understands that coordinate systems provide convenient and efficient ways of representing geometric figures and uses them accordingly (G.7). The student is expected to

Working with Coordinate Planes

- use whole numbers to locate and name points on a number line (2)
- locate and name points on a number line using whole numbers and fractions, including halves and fourths (3)
- locate and name points on a number line using whole numbers, fractions such as halves and fourths, and decimals such as tenths (4)
- locate and name points on a coordinate grid using ordered pairs of whole numbers (5)
- locate and name points on a coordinate plane using ordered pairs of non-negative rational numbers (6)
- locate and name points on a coordinate plane using ordered pairs of integers (7)
- graph reflections across the horizontal or vertical axis and graph translations on a coordinate plane (7)
- locate and name points on a coordinate plane using ordered pairs of rational numbers (8)
- use one- and two-dimensional coordinate systems to represent points, lines, rays, line segments, and figures (Geom)
- use slopes and equations of lines to investigate geometric relationships, including parallel lines, perpendicular lines, and special segments of triangles and other polygons (Geom)
- derive and use formulas involving length, slope, and midpoint (Geom)

Geometry and spatial reasoning. The student uses geometry to model and describe the physical world (7.8; 8.7). The student uses transformational geometry to develop spatial sense (8.6).

Geometric structure. The student understands the structure of, and relationships within, an axiomatic system (G.1).

Geometric patterns. The student uses a variety of representations to describe geometric relationships and solve problems (G.5).

Dimensionality and the geometry of location. The student analyzes the relationship between three-dimensional geometric figures and related two-dimensional representations and uses these representations to solve problems (G.6).

Congruence and the geometry of size. The student uses tools to determine measurements of geometric figures and extends measurement concepts to find perimeter, area, and volume in problem situations (G.8).

Similarity and the geometry of shape. The student applies the concepts of similarity to justify properties of figures and solve problems (G.11).

The student is expected to

Geometric Properties and Relationships

- sketch three-dimensional figures when given the top, side, and front views (7)
- make a net (two-dimensional model) of the surface area of a three-dimensional figure (7)
- use geometric concepts and properties to solve problems in fields such as art and architecture (7-8)
- generate similar figures using dilations including enlargements and reductions (8)
- graph dilations, reflections, and translations on a coordinate plane (8)
- draw three-dimensional figures from different perspectives (8)
- use pictures or models to demonstrate the Pythagorean Theorem (8)
- develop an awareness of the structure of a mathematical system, connecting definitions, postulates, logical reasoning, and theorems (Geom)

- recognize the historical development of geometric systems and know mathematics is developed for a variety of purposes (Geom)
- compare and contrast the structures and implications of Euclidean and non-Euclidean geometries (Geom)
- use numeric and geometric patterns to develop algebraic expressions representing geometric properties (Geom)
- use numeric and geometric patterns to make generalizations about geometric properties, including properties of polygons, ratios in similar figures and solids, and angle relationships in polygons and circles (Geom)
- use properties of transformations and their compositions to make connections between mathematics and the real world, such as tessellations (Geom)
- identify and apply patterns from right triangles to solve meaningful problems, including special right triangles (45-45-90 and 30-60-90) and triangles whose sides are Pythagorean triples (Geom)
- describe and draw the intersection of a given plane with various three-dimensional geometric figures (Geom)
- use nets to represent and construct three-dimensional geometric figures (Geom)
- use orthographic and isometric views of three-dimensional geometric figures to represent and construct three-dimensional geometric figures and solve problems (Geom)
- derive, extend, and use the Pythagorean Theorem (Geom)
- use and extend similarity properties and transformations to explore and justify conjectures about geometric figures (Geom)
- use ratios to solve problems involving similar figures (Geom)
- develop, apply, and justify triangle similarity relationships, such as right triangle ratios, trigonometric ratios, and Pythagorean triples using a variety of methods (Geom)
- describe the effect on perimeter, area, and volume when one or more dimensions of a figure are changed and apply this idea in solving problems (Geom)

Measurement

Measurement skills. The student verbally describes or demonstrates attributes of persons or objects, such as length, area, capacity, or weight (Pre-K.V.D).

Measurement. The student directly compares the attributes of length, area, weight/mass, capacity, and/or relative temperature. The student uses comparative language to solve problems and answer questions (K.10). The student directly compares the attributes of length, area, weight/mass, capacity, and temperature. The student uses comparative language to solve problems and answer questions. The student selects and uses nonstandard units to describe length (1.7). The student directly compares the attributes of length, area, weight/mass, and capacity, and uses comparative language to solve problems and answer questions. The student selects and uses nonstandard units to describe length, area, capacity, and weight/mass. The student recognizes and uses models that approximate standard units (from both SI, also known as metric, and customary systems) of length, weight/mass, capacity, and time (2.9). The student directly compares the attributes of length, area, weight/mass, and capacity, and uses comparative language to solve problems and answer questions. The student selects and uses standard units to describe length, area, capacity/volume, and weight/mass (3.11). The student applies measurement concepts. The student is expected to estimate and measure to solve problems involving length (including perimeter) and area. The student uses measurement tools to measure capacity/volume and

weight/mass (4.11). The student applies measurement concepts involving length (including perimeter), area, capacity/volume, and weight/mass to solve problems (5.10). The student solves application problems involving estimation and measurement of length, area, time, temperature, volume, weight, and angles (6.8). The student solves application problems involving estimation and measurement (7.9). The student uses procedures to determine measures of three-dimensional figures (8.8). The student uses indirect measurement to solve problems (8.9). The student describes how changes in dimensions affect linear, area, and volume measures (8.10).

Congruence and the geometry of size. The student uses tools to determine measurements of geometric figures and extends measurement concepts to find perimeter, area, and volume in problem situations (G.8). The student is expected to

Comparisons

- recognize and compare heights or lengths of people or objects (Pre-K)
- informally recognize and compare weights of objects or people (Pre-K)
- compare and order two or three concrete objects according to length (longer/shorter than, or the same) (K)
- compare the areas of two flat surfaces of two-dimensional figures (covers more, covers less, or covers the same) (K)
- compare two containers according to capacity (holds more, holds less, or holds the same) (K)
- compare two objects according to weight/mass (heavier than, lighter than or equal to) (K)
- compare and order two or more concrete objects according to length (from longest to shortest) (1)
- describe the relationship between the size of the unit and the number of units needed to measure the length of an object (1)
- compare and order the area of two or more two-dimensional surfaces (from covers the most to covers the least) (1)
- compare and order two or more containers according to capacity (from holds the most to holds the least) (1)
- compare and order two or more objects according to weight/mass (from heaviest to lightest) (1)
- explain the difference between weight and mass (4)
- perform simple conversions within the same measurement system (SI (metric) or customary) (5)
- convert measures within the same measurement system (customary and metric) based on relationships between units (6)
- use conversions between measurement systems to solve problems in real-world situations (Geom)

Using Models

- recognize how much can be placed within an object (Pre-K)
- estimate and measure length using nonstandard units such as paper clips or sides of color tiles (1)
- identify concrete models that approximate standard units of length and use them to measure length (2)
- select a non-standard unit of measure such as square tiles to determine the area of a two-dimensional surface (2)
- select a non-standard unit of measure such as a bathroom cup or a jar to determine the capacity of a given container (2)

- select a non-standard unit of measure such as beans or marbles to determine the weight/mass of a given object (2)
- use linear measurement tools to estimate and measure lengths using standard units (3)
- use standard units to find the perimeter of a shape (3)
- use concrete and pictorial models of square units to determine the area of two-dimensional surfaces (3)
- identify concrete models that approximate standard units of weight/mass and use them to measure weight/mass (3)
- identify concrete models that approximate standard units for capacity and use them to measure capacity (3)
- use concrete models that approximate cubic units to determine the volume of a given container or other three-dimensional geometric figure (3)
- estimate and use measurement tools to determine length (including perimeter), area, capacity, and weight/mass using standard units SI (metric) and customary (4)
- perform simple conversions between different units of length, between different units of capacity, and between different units of weight within the customary measurement system (4)
- use concrete models of standard cubic units to measure volume (4)
- estimate volume in cubic units (4)
- connect models for perimeter, area, and volume with their respective formulas (5)
- select and use appropriate units and formulas to measure length, perimeter, area, and volume (5)
- estimate measurements (including circumference) and evaluate reasonableness of results (6)
- select and use appropriate units, tools, or formulas to measure and to solve problems involving length (including perimeter), area, time, temperature, volume, and weight (6)
- measure angles (6)
- estimate measurements and solve application problems involving length (including perimeter and circumference) and area of polygons and other shapes (7)
- connect models for volume of prisms (triangular and rectangular) and cylinders to formulas of prisms (triangular and rectangular) and cylinders (7)
- estimate measurements and solve application problems involving volume of prisms (rectangular and triangular) and cylinders (7)
- find lateral and total surface area of prisms, pyramids, and cylinders using concrete models and nets (two-dimensional models) (8)
- connect models of prisms, cylinders, pyramids, spheres, and cones to formulas for volume of these objects (8)
- estimate measurements and use formulas to solve application problems involving lateral and total surface area and volume (8)
- use the Pythagorean Theorem to solve real-life problems (8)
- use proportional relationships in similar two-dimensional figures or similar three-dimensional figures to find missing measurements (8)
- describe the resulting effects on perimeter and area when dimensions of a shape are changed proportionally (8)
- describe the resulting effect on volume when dimensions of a solid are changed proportionally (8)
- find areas of regular polygons, circles, and composite figures (Geom)
- find areas of sectors and arc lengths of circles using proportional reasoning (Geom)

- find surface areas and volumes of prisms, pyramids, spheres, cones, cylinders, and composites of these figures in problem situations (Geom)
- use area models to connect geometry to probability and statistics (Geom)

Measurement skills. The student verbally describes or demonstrates attributes of persons or objects, such as length, area, capacity, or weight (Pre-K.V.D).

Measurement. The student directly compares the attributes of length, area, weight/mass, capacity, and/or relative temperature. The student uses comparative language to solve problems and answer questions (K.10). The student uses time to describe, compare and order events, and situations (K.11). The student directly compares the attributes of length, area, weight/mass, capacity, and temperature. The student uses comparative language to solve problems and answer questions. The student selects and uses nonstandard units to describe length (1.7). The student understands that time can be measured. The student uses time to describe and compare situations (1.8). The student uses standard tools to estimate and measure time and temperature (in degrees Fahrenheit) (2.10). The student reads and writes time and measures temperature in degrees Fahrenheit to solve problems (3.12). The student applies measurement concepts. The student measures time and temperature (in degrees Fahrenheit and Celsius) (4.12; 5.11). The student is expected to

Time and Temperature

- use language to describe concepts associated with the passing of time (Pre-K)
- compare situations or objects according to relative temperature (hotter/colder than, or the same as) (K)
- compare events according to duration such as more time than or less time than (K)
- sequence events (up to three) (K)
- read a calendar using days, weeks, and months (K)
- compare and order two or more objects according to relative temperature (from hottest to coldest) (1)
- order three or more events according to duration (1)
- read time to the hour and half-hour using analog and digital clocks (1)
- read a thermometer to gather data (2)
- read and write times shown on analog and digital clocks using five-minute increments (2)
- describe activities that take approximately one second, one minute, and one hour (2)
- use a thermometer to measure temperature (3)
- tell and write time shown on analog and digital clocks (3)
- use a thermometer to measure temperature and changes in temperature (4)
- use tools such as a clock with gears or a stopwatch to solve problems involving elapsed time (4)
- solve problems involving changes in temperature (5)
- solve problems involving elapsed time (5)

Probability and Statistics

Classification and patterns skills. The student sorts and classifies objects using one or more attributes and uses attributes of objects to duplicate and create patterns (Pre-K.V.E).

Probability and statistics. The student constructs and uses graphs of real objects or pictures to answer questions (K.12). The student displays data in an organized form (1.9). The student uses information from organized data (1.10). The student organizes data to make it useful for interpreting information (2.11). The student solves problems by collecting, organizing, displaying, and interpreting sets of data (3.13; 4.13; 5.13). The student uses statistical representations to analyze data (6.10). The student understands that the way a set of data is displayed influences its interpretation (7.11). The student uses statistical procedures to describe data (8.12). The student evaluates predictions and conclusions based on statistical data (8.13). The student is expected to

Working with Data

- sort objects that are the same and different into groups and use language to describe how the groups are similar and different (Pre-K)
- collect data and organize it in a graphic representation (Pre-K)
- construct graphs using real objects or pictures in order to answer questions (K)
- use information from a graph of real objects or pictures in order to answer questions (K)
- collect and sort data (1)
- use organized data to construct real-object graphs, picture graphs, and bar-type graphs (1)
- draw conclusions and answer questions using information organized in real-object graphs, picture graphs, and bar-type graphs (1)
- identify events as certain or impossible such as drawing a red crayon from a bag of green crayons (1)
- construct picture graphs and bar-type graphs (2)
- draw conclusions and answer questions based on picture graphs and bar-type graphs (2)
- use data to describe events as more likely or less likely such as drawing a certain color crayon from a bag of seven red crayons and three green crayons (2)
- collect, organize, record, and display data in pictographs and bar graphs where each picture or cell might represent more than one piece of data (3)
- interpret information from pictographs and bar graphs (3)
- use data to describe events as more likely than, less likely than, or equally likely as (3)
- use concrete objects or pictures to make generalizations about determining all possible combinations of a given set of data or of objects in a problem situation (4)
- interpret bar graphs (4)
- use tables of related number pairs to make line graphs (5)
- graph a given set of data using an appropriate graphical representation such as a picture or line graph (5)
- select and use an appropriate representation for presenting and displaying different graphical representations of the same data including line plot, line graph, bar graph, and stem and leaf plot (6)
- sketch circle graphs to display data (6)
- solve problems by collecting, organizing, displaying, and interpreting data (6)
- select and use an appropriate representation for presenting and displaying relationships among collected data, including line plot, line graph, bar graph, stem and leaf plot, circle graph, and Venn diagrams, and justify the selection (7)

- make inferences and convincing arguments based on an analysis of given or collected data (7)
- draw conclusions and make predictions by analyzing trends in scatterplots (8)
- select and use an appropriate representation for presenting and displaying relationships among collected data, including line plots, line graphs, stem and leaf plots, circle graphs, bar graphs, box and whisker plots, histograms, and Venn diagrams, with and without the use of technology (8)
- evaluate methods of sampling to determine validity of an inference made from a set of data (8)
- recognize misuses of graphical or numerical information and evaluate predictions and conclusions based on data analysis (8)

Probability and statistics. The student describes and predicts the results of a probability experiment (5.12). The student uses experimental and theoretical probability to make predictions (6.9). The student recognizes that a physical or mathematical model (including geometric) can be used to describe the experimental and theoretical probability of real-life events (7.10). The student applies concepts of theoretical and experimental probability to make predictions (8.11). The student is expected to

Probability

- use fractions to describe the results of an experiment (5)
- use experimental results to make predictions (5)
- list all possible outcomes of a probability experiment such as tossing a coin (5)
- construct sample spaces using lists and tree diagrams (6)
- find the probabilities of a simple event and its complement and describe the relationship between the two (6)
- construct sample spaces for simple or composite experiments (7)
- find the probability of independent events (7)
- find the probabilities of dependent and independent events (8)
- use theoretical probabilities and experimental results to make predictions and decisions (8)
- select and use different models to simulate an event (8)

Probability and statistics. The student solves problems by collecting, organizing, displaying, and interpreting sets of data (5.13). The student uses statistical representations to analyze data (6.10). The student uses measures of central tendency and variability to describe a set of data (7.12). The student uses statistical procedures to describe data (8.12). The student is expected to

Measures of Central Tendency

- describe characteristics of data presented in tables and graphs including median, mode, and range (5)
- identify mean (using concrete objects and pictorial models), median, mode, and range of a set of data (6)
- describe a set of data using mean, median, mode, and range (7)
- choose among mean, median, mode, or range to describe a set of data and justify the choice for a particular situation (7)

- use variability (range, including interquartile range (IQR)) and select the appropriate measure of central tendency to describe a set of data and justify the choice for a particular situation (8)

Underlying Processes and Mathematical Tools

Underlying processes and mathematical tools. The student applies [Kindergarten; Grade 1; Grade 2; Grade 3; Grade 4; Grade 5] mathematics to solve problems connected to everyday experiences and activities in and outside of school (K.13; 1.11; 2.12; 3.14; 4.14; 5.14). The student applies [Grade 6; Grade 7; Grade 8] mathematics to solve problems connected to everyday experiences, investigations in other disciplines, and activities in and outside of school (6.11; 7.13; 8.14). The student is expected to

Solving Problems

- select or develop an appropriate problem-solving strategy including drawing a picture, looking for a pattern, systematic guessing and checking, or acting it out in order to solve a problem (K)
- solve problems with guidance that incorporates the processes of understanding the problem, making a plan, carrying out the plan, and evaluating the solution for reasonableness (K–2)
- identify mathematics in everyday situations (K–5)
- use tools such as real objects, manipulatives, and technology to solve problems (K–5)
- select or develop an appropriate problem-solving plan or strategy including drawing a picture, looking for a pattern, systematic guessing and checking, or acting it out in order to solve a problem (1–2)
- solve problems that incorporate understanding the problem, making a plan, carrying out the plan, and evaluating the solution for reasonableness (3–5)
- select or develop an appropriate problem-solving plan or strategy, including drawing a picture, looking for a pattern, systematic guessing and checking, acting it out, making a table, working a simpler problem, or working backwards to solve a problem (3–5)
- identify and apply mathematics to everyday experiences, to activities in and outside of school, with other disciplines, and with other mathematical topics (6–8)
- use a problem-solving model that incorporates understanding the problem, making a plan, carrying out the plan, and evaluating the solution for reasonableness (6–8)
- select or develop an appropriate problem-solving strategy from a variety of different types, including drawing a picture, looking for a pattern, systematic guessing and checking, acting it out, making a table, working a simpler problem, or working backwards to solve a problem (6–8)
- select tools such as real objects, manipulatives, paper/pencil, and technology or techniques such as mental math, estimation, and number sense to solve problems (6–8)

Underlying processes and mathematical tools. The student communicates about [Kindergarten; Grade 1; Grade 2; Grade 3; Grade 4; Grade 5] mathematics using informal language (K.14; 1.12; 2.13; 3.15; 4.15; 5.15). The student communicates about [Grade 6; Grade 7; Grade 8]

mathematics through informal and mathematical language, representations, and models (6.12; 7.14; 8.15). The student is expected to

Mathematics and Symbols

- communicate mathematical ideas using objects, words, pictures, numbers, and technology (K)
- relate everyday language to mathematical language and symbols (K)
- explain and record observations using objects, words, pictures, numbers, and technology (1–5)
- relate informal language to mathematical language and symbols (1–5)
- communicate mathematical ideas using language, efficient tools, appropriate units, and graphical, numerical, physical, or algebraic mathematical models (6–8)
- evaluate the effectiveness of different representations to communicate ideas (6–8)

Underlying processes and mathematical tools. The student uses logical reasoning (K.15; 1.13; 2.14; 3.16; 4.16; 5.16). The student uses logical reasoning to make conjectures and verify conclusions (6.13; 7.15; 8.16). The student is expected to

Mathematical Reasoning

- justify his or her thinking using objects, words, pictures, numbers, and technology (K–2)
- make generalizations from patterns or sets of examples and nonexamples (3–5)
- justify why an answer is reasonable and explain the solution process (3–5)
- make conjectures from patterns or sets of examples and nonexamples (6–8)
- validate his/her conclusions using mathematical properties and relationships (6–8)