

ATTACHMENT II
Text of Proposed Revisions to 19 TAC

Chapter 111. Texas Essential Knowledge and Skills for Mathematics

Subchapter A. Elementary

§111.1. Implementation of Texas Essential Knowledge and Skills for Mathematics, Elementary, Beginning with School Year 2013-2014.

The provisions of §§111.2-111.7 of this subchapter shall be implemented by school districts beginning with the 2013-2014 school year. This implementation date shall supersede any other implementation dates found in this subchapter.

§111.2. Kindergarten, Beginning with School Year 2013-2014.

(a) Introduction.

- (1) The desire to achieve educational excellence is the driving force behind the Texas essential knowledge and skills for mathematics, guided by the college and career readiness standards. By embedding statistics, probability, and finance, while focusing on fluency and solid understanding, Texas will lead the way in mathematics education and prepare all Texas students for the challenges they will face in the 21st century.
- (2) The process standards describe ways in which students are expected to engage in the content. The placement of the process standards at the beginning of the knowledge and skills listed for each grade and course is intentional. The process standards weave the other knowledge and skills together so that students may be successful problem solvers and use mathematics efficiently and effectively in daily life. The process standards are integrated at every grade level and course. When possible, students will apply mathematics to problems arising in everyday life, society, and the workplace. Students will use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution. Students will select appropriate tools such as real objects, manipulatives, paper and pencil, and technology and techniques such as mental math, estimation, and number sense to solve problems. Students will effectively communicate mathematical ideas, reasoning, and their implications using multiple representations such as symbols, diagrams, graphs, and language. Students will use mathematical relationships to generate solutions and make connections and predictions. Students will analyze mathematical relationships to connect and communicate mathematical ideas. Students will display, explain, or justify mathematical ideas and arguments using precise mathematical language in written or oral communication.
- (3) The primary focal areas in Kindergarten are understanding counting and cardinality, understanding addition as joining and subtraction as separating, and comparing objects by measureable attributes.
 - (A) Students develop number and operations through several fundamental concepts. Students know number names and the counting sequence. Counting and cardinality lay a solid foundation for number. Students apply the principles of counting to make the connection between numbers and quantities.
 - (B) Students use meanings of numbers to create strategies for solving problems and responding to practical situations involving addition and subtraction.
 - (C) Students identify characteristics of objects that can be measured and directly compare objects according to these measureable attributes.
- (4) Statements that contain the word "including" reference content that must be mastered, while those containing the phrase "such as" are intended as possible illustrative examples.

(b) Knowledge and skills.

- (1) Mathematical process standards. The student uses mathematical processes to acquire and demonstrate mathematical understanding. The student is expected to:
 - (A) apply mathematics to problems arising in everyday life, society, and the workplace;
 - (B) use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution;
 - (C) select tools, including real objects, manipulatives, paper and pencil, and technology as appropriate, and techniques, including mental math, estimation, and number sense as appropriate, to solve problems;
 - (D) communicate mathematical ideas, reasoning, and their implications using multiple representations, including symbols, diagrams, graphs, and language as appropriate;
 - (E) create and use representations to organize, record, and communicate mathematical ideas;
 - (F) analyze mathematical relationships to connect and communicate mathematical ideas; and
 - (G) display, explain, and justify mathematical ideas and arguments using precise mathematical language in written or oral communication.
- (2) Number and operations. The student applies mathematical process standards to understand how to represent and compare whole numbers, the relative position and magnitude of whole numbers, and relationships within the numeration system. The student is expected to:
 - (A) count forward and backward to at least 20 with and without objects;
 - (B) read, write, and represent whole numbers from 0 to at least 20 with and without objects or pictures;
 - (C) count a set of objects up to at least 20 and demonstrate that the last number said tells the number of objects in the set regardless of their arrangement;
 - (D) recognize instantly the quantity of a small group of objects in organized and random arrangements;
 - (E) generate a set using concrete and pictorial models that represents a number that is more than, less than, and equal to a given number up to 20;
 - (F) generate a number that is one more than or one less than another number up to at least 20;
 - (G) compare sets of objects up to at least 20 in each set using comparative language;
 - (H) use comparative language to describe two numbers up to 20 presented as written numerals; and
 - (I) compose and decompose numbers up to 10 with objects and pictures.
- (3) Number and operations. The student applies mathematical process standards to develop an understanding of addition and subtraction situations in order to solve problems. The student is expected to:
 - (A) model the action of joining to represent addition and the action of separating to represent subtraction;
 - (B) solve word problems using objects and drawings to find sums up to 10 and differences within 10; and
 - (C) explain the strategies used to solve problems involving adding and subtracting within 10 using spoken words, concrete and pictorial models, and number sentences.
- (4) Number and operations. The student applies mathematical process standards to identify coins in order to recognize the need for monetary transactions. The student is expected to identify U.S. coins by name, including pennies, nickels, dimes, and quarters.

- (5) Algebraic reasoning. The student applies mathematical process standards to identify the pattern in the number word list. The student is expected to:
- (A) recite numbers up to at least 100 by ones beginning with any given number; and
 - (B) represent addition and subtraction with objects, drawings, situations, verbal explanations, or number sentences.
- (6) Geometry and measurement. The student applies mathematical process standards to analyze attributes of two-dimensional shapes and three-dimensional solids to develop generalizations about their properties. The student is expected to:
- (A) identify two-dimensional shapes, including circles, triangles, rectangles, and squares as special rectangles;
 - (B) identify three-dimensional solids, including cylinders, cones, spheres, and cubes, in the real world;
 - (C) identify two-dimensional components of three-dimensional objects such as the face of a tissue box is a rectangle;
 - (D) identify attributes of two-dimensional shapes using informal and formal geometric language interchangeably such as number of corners or vertices and number of sides;
 - (E) classify and sort a variety of regular and irregular two- and three-dimensional figures regardless of orientation or size; and
 - (F) create two-dimensional shapes using a variety of materials and drawings.
- (7) Geometry and measurement. The student applies mathematical process standards to directly compare measureable attributes. The student is expected to:
- (A) give an example of a measurable attribute of a given object, including length, capacity, and weight; and
 - (B) compare two objects with a common measureable attribute to see which object has more of/less of the attribute and describe the difference.
- (8) Data analysis. The student applies mathematical process standards to collect and organize data to make it useful for interpreting information. The student is expected to:
- (A) collect, sort, and organize data into two or three categories;
 - (B) use data to create real-object and picture graphs; and
 - (C) draw conclusions from real-object and picture graphs.

§111.3. Grade 1, Beginning with School Year 2013-2014.

(a) Introduction.

- (1) The desire to achieve educational excellence is the driving force behind the Texas essential knowledge and skills for mathematics, guided by the college and career readiness standards. By embedding statistics, probability, and finance, while focusing on fluency and solid understanding, Texas will lead the way in mathematics education and prepare all Texas students for the challenges they will face in the 21st century.
- (2) The process standards describe ways in which students are expected to engage in the content. The placement of the process standards at the beginning of the knowledge and skills listed for each grade and course is intentional. The process standards weave the other knowledge and skills together so that students may be successful problem solvers and use mathematics efficiently and effectively in daily life. The process standards are integrated at every grade level and course. When possible, students will apply mathematics to problems arising in everyday life, society, and the workplace. Students will use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and

evaluating the problem-solving process and the reasonableness of the solution. Students will select appropriate tools such as real objects, manipulatives, paper and pencil, and technology and techniques such as mental math, estimation, and number sense to solve problems. Students will effectively communicate mathematical ideas, reasoning, and their implications using multiple representations such as symbols, diagrams, graphs, and language. Students will use mathematical relationships to generate solutions and make connections and predictions. Students will analyze mathematical relationships to connect and communicate mathematical ideas. Students will display, explain, or justify mathematical ideas and arguments using precise mathematical language in written or oral communication.

(3) The primary focal areas in Grade 1 are understanding and applying place value, solving problems involving addition and subtraction, and composing and decomposing two-dimensional shapes and three-dimensional solids.

(A) Students use relationships within the numeration system to understand the sequential order of the counting numbers and their relative magnitude.

(B) Students extend their use of addition and subtraction beyond the actions of joining and separating to include comparing and combining. Students use properties of operations and the relationship between addition and subtraction to solve problems. By comparing a variety of solution strategies, students use efficient, accurate, and generalizable methods to perform operations.

(C) Students use basic shapes and spatial reasoning to model objects in their environment and construct more complex shapes. Students are able to identify, name, and describe basic two-dimensional shapes and three-dimensional solids.

(4) Statements that contain the word "including" reference content that must be mastered, while those containing the phrase "such as" are intended as possible illustrative examples.

(b) Knowledge and skills.

(1) Mathematical process standards. The student uses mathematical processes to acquire and demonstrate mathematical understanding. The student is expected to:

(A) apply mathematics to problems arising in everyday life, society, and the workplace;

(B) use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution;

(C) select tools, including real objects, manipulatives, paper and pencil, and technology as appropriate, and techniques, including mental math, estimation, and number sense as appropriate, to solve problems;

(D) communicate mathematical ideas, reasoning, and their implications using multiple representations, including symbols, diagrams, graphs, and language as appropriate;

(E) create and use representations to organize, record, and communicate mathematical ideas;

(F) analyze mathematical relationships to connect and communicate mathematical ideas; and

(G) display, explain, and justify mathematical ideas and arguments using precise mathematical language in written or oral communication.

(2) Number and operations. The student applies mathematical process standards to represent and compare whole numbers, the relative position and magnitude of whole numbers, and relationships within the numeration system related to place value. The student is expected to:

(A) recognize instantly the quantity of structured arrangements such as seen on a die or a ten frame;

(B) use concrete and pictorial models to compose and decompose numbers up to 120 in more than one way as so many hundreds, so many tens, and so many ones;

- (C) use objects, pictures, and expanded and standard forms to represent numbers up to 120;
 - (D) generate a number that is greater than or less than a given whole number up to 120;
 - (E) use place value to compare whole numbers up to 120 using comparative language; and
 - (F) order whole numbers up to 120 using place value and open number lines.
- (3) Number and operations. The student applies mathematical process standards to develop and use strategies for whole number addition and subtraction computations in order to solve problems. The student is expected to:
- (A) use concrete and pictorial models to determine the sum of a multiple of 10 and a one-digit number in problems up to 99;
 - (B) use objects and pictorial models to solve word problems involving joining, separating, and comparing sets within 20 and unknowns as any one of the terms in the problem such as $2 + 4 = \square$; $3 + \square = 7$; and $5 = \square - 3$;
 - (C) compose 10 with two or more addends with and without concrete objects;
 - (D) apply basic fact strategies to add and subtract within 20, including making 10 and decomposing a number leading to a 10;
 - (E) explain strategies used to solve addition and subtraction problems up to 20 using spoken words, objects, pictorial models, and number sentences; and
 - (F) generate and solve problem situations when given a number sentence involving addition and subtraction of numbers within 20.
- (4) Number and operations. The student applies mathematical process standards to identify coins, their values, and the relationships among them in order to recognize the need for monetary transactions. The student is expected to:
- (A) identify U.S. coins, including pennies, nickels, dimes, and quarters, by value and describe the relationships among them;
 - (B) write a number with the cent symbol to describe the value of a coin; and
 - (C) use relationships to count by twos, fives, and tens to determine the value of pennies, nickels, and dimes.
- (5) Algebraic reasoning. The student applies mathematical process standards to identify and apply number patterns within properties of numbers and operations in order to describe relationships. The student is expected to:
- (A) recite numbers forward and backward from any given number between 1 and 120;
 - (B) skip count by twos, fives, and tens to 100;
 - (C) skip count by twos, fives, and tens to determine the total number of objects up to 120 in a set;
 - (D) use relationships to determine the number that is 10 more and 10 less than a given number up to 120;
 - (E) represent word problems involving addition and subtraction of whole numbers up to 20 using concrete and pictorial models and number sentences;
 - (F) understand that the equal sign represents a relationship where statements on each side of the equal sign are true;
 - (G) determine the unknown whole number in an addition or subtraction equation when the unknown may be any one of the three or four terms in the equation;
 - (H) identify relationships between addition facts and related subtraction sentences such as $3 + 2 = 5$ and $5 - 2 = 3$; and

- (I) apply properties of operations as strategies to add and subtract such as if $2 + 3 = 5$ is known, then $3 + 2 = 5$.
- (6) Geometry and measurement. The student applies mathematical process standards to analyze attributes of two-dimensional shapes and three-dimensional solids to develop generalizations about their properties. The student is expected to:
- (A) classify and sort regular and irregular two-dimensional shapes based on attributes using informal geometric language;
 - (B) distinguish between attributes that define a two-dimensional or three-dimensional figure such as a closed figure with three sides is a triangle or a solid with exactly six rectangular faces is a rectangular prism and attributes that do not define the shape such as orientation or color;
 - (C) create two-dimensional figures, including circles, triangles, rectangles, and squares, as special rectangles, rhombuses, and hexagons;
 - (D) identify two-dimensional shapes, including circles, triangles, rectangles, and squares, as special rectangles, rhombuses, and hexagons and describe their attributes using formal geometric language such as vertex and side;
 - (E) identify three-dimensional solids, including spheres, cones, cylinders, rectangular prisms (including cubes), and triangular prisms, and describe their attributes using formal geometric language such as vertex, edge, and face;
 - (F) compose two-dimensional shapes by joining two, three, or four figures to produce a target shape in more than one way if possible;
 - (G) partition two-dimensional figures such as circles and rectangles into two and four fair shares or equal parts and describe the parts using words such as "halves," "half of," "fourths," or "quarters"; and
 - (H) identify examples and non-examples of halves and fourths.
- (7) Geometry and measurement. The student applies mathematical process standards to select and use units to describe length and time. The student is expected to:
- (A) use measuring tools such as adding machine tape, ribbon, or string to measure the length of objects to reinforce the continuous nature of linear measurement;
 - (B) demonstrate that the length of an object is the number of same-size units of length that, when laid end-to-end with no gaps or overlaps, reach from one end of the object to the other;
 - (C) measure the same object/distance with units of two different lengths and describe how and why the measurements differ;
 - (D) describe a length to the nearest whole unit using a number and a unit such as five craft sticks; and
 - (E) tell time to the hour and half hour using analog and digital clocks.
- (8) Data analysis. The student applies mathematical process standards to organize data to make it useful for interpreting information and solving problems. The student is expected to:
- (A) collect, sort, and organize data in up to three categories using models/representations such as tally marks or T-charts;
 - (B) use data to create picture and bar-type graphs; and
 - (C) draw conclusions and generate and answer questions using information from picture and bar-type graphs.

§111.4. Grade 2, Beginning with School Year 2013-2014.

(a) Introduction.

- (1) The desire to achieve educational excellence is the driving force behind the Texas essential knowledge and skills for mathematics, guided by the college and career readiness standards. By embedding statistics, probability, and finance, while focusing on fluency and solid understanding, Texas will lead the way in mathematics education and prepare all Texas students for the challenges they will face in the 21st century.**
- (2) The process standards describe ways in which students are expected to engage in the content. The placement of the process standards at the beginning of the knowledge and skills listed for each grade and course is intentional. The process standards weave the other knowledge and skills together so that students may be successful problem solvers and use mathematics efficiently and effectively in daily life. The process standards are integrated at every grade level and course. When possible, students will apply mathematics to problems arising in everyday life, society, and the workplace. Students will use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution. Students will select appropriate tools such as real objects, manipulatives, paper and pencil, and technology and techniques such as mental math, estimation, and number sense to solve problems. Students will effectively communicate mathematical ideas, reasoning, and their implications using multiple representations such as symbols, diagrams, graphs, and language. Students will use mathematical relationships to generate solutions and make connections and predictions. Students will analyze mathematical relationships to connect and communicate mathematical ideas. Students will display, explain, or justify mathematical ideas and arguments using precise mathematical language in written or oral communication.**
- (3) The primary focal areas in Grade 2 are making comparisons within the base-10 place value system, solving problems with addition and subtraction within 100, and building foundations for multiplication.**
 - (A) Students develop an understanding of the base-10 place value system and place value concepts. The students' understanding of base-10 place value includes ideas of counting in units and multiples of thousands, hundreds, tens, and ones and a grasp of number relationships, which students demonstrate in a variety of ways.**
 - (B) Students identify situations in which addition and subtraction are useful to solve problems. Students develop a variety of strategies to use efficient, accurate, and generalizable methods to add and subtract multi-digit whole numbers.**
 - (C) Students use the relationship between skip counting and equal groups of objects to represent the addition or subtraction of equivalent sets, which builds a strong foundation for multiplication and division.**
- (4) Statements that contain the word "including" reference content that must be mastered, while those containing the phrase "such as" are intended as possible illustrative examples.**

(b) Knowledge and skills.

- (1) Mathematical process standards. The student uses mathematical processes to acquire and demonstrate mathematical understanding. The student is expected to:**
 - (A) apply mathematics to problems arising in everyday life, society, and the workplace;**
 - (B) use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution;**
 - (C) select tools, including real objects, manipulatives, paper and pencil, and technology as appropriate, and techniques, including mental math, estimation, and number sense as appropriate, to solve problems;**

- (D) communicate mathematical ideas, reasoning, and their implications using multiple representations, including symbols, diagrams, graphs, and language as appropriate;
 - (E) create and use representations to organize, record, and communicate mathematical ideas;
 - (F) analyze mathematical relationships to connect and communicate mathematical ideas; and
 - (G) display, explain, and justify mathematical ideas and arguments using precise mathematical language in written or oral communication.
- (2) Number and operations. The student applies mathematical process standards to understand how to represent and compare whole numbers, the relative position and magnitude of whole numbers, and relationships within the numeration system related to place value. The student is expected to:
- (A) use concrete and pictorial models to compose and decompose numbers up to 1,200 in more than one way as a sum of so many thousands, hundreds, tens, and ones;
 - (B) use standard, word, and expanded forms to represent numbers up to 1,200;
 - (C) generate a number that is greater than or less than a given whole number up to 1,200;
 - (D) use place value to compare whole numbers up to 1,200 using comparative language, numbers, and symbols ($>$, $<$, or $=$);
 - (E) locate the position of a given whole number on an open number line;
 - (F) name the whole number that corresponds to a specific point on a number line; and
 - (G) order whole numbers up to 1,200 using place value and open number lines.
- (3) Number and operations. The student applies mathematical process standards to recognize and represent fractional units and communicates how they are used to name parts of a whole. The student is expected to:
- (A) partition objects such as strips, lines, regular polygons, and circles into equal parts and name the parts, including halves, fourths, and eighths, using words such as "one-half" or "three-fourths";
 - (B) explain that the more fractional parts used to make a whole, the smaller the part; and the fewer the fractional parts, the larger the part;
 - (C) use concrete models to count fractional parts beyond one whole using words such as "one-fourth," "two-fourths," "three-fourths," "four-fourths," "five-fourths," or "one and one-fourth," and recognize how many parts it takes to equal one whole such as four-fourths equals one whole; and
 - (D) identify examples and non-examples of halves, fourths, and eighths.
- (4) Number and operations. The student applies mathematical process standards to develop and use strategies and methods for whole number computations in order to solve addition and subtraction problems with efficiency and accuracy. The student is expected to:
- (A) recall basic facts to add and subtract within 20 with automaticity;
 - (B) use mental strategies, flexible methods, and algorithms based on knowledge of place value and equality to add and subtract two-digit numbers;
 - (C) solve one-step and multi-step word problems involving addition and subtraction of two-digit numbers using a variety of strategies based on place value, including algorithms; and
 - (D) generate and solve problem situations for a given mathematical number sentence involving addition and subtraction of whole numbers within 100.
- (5) Number and operations. The student applies mathematical process standards to determine the value of coins in order to solve monetary transactions. The student is expected to:

- (A) determine the value of a collection of coins up to one dollar; and
 - (B) use the cent symbol, dollar sign, and the decimal point to name the value of a collection of coins.
- (6) Number and operations. The student applies mathematical process standards to connect repeated addition and subtraction to multiplication and division situations that involve equal groupings and shares. The student is expected to:
- (A) model, create, and describe contextual multiplication situations in which equivalent sets of concrete objects are joined; and
 - (B) model, create, and describe contextual division situations in which a set of concrete objects is separated into equivalent sets.
- (7) Algebraic reasoning. The student applies mathematical process standards to identify and apply number patterns within properties of numbers and operations in order to describe relationships. The student is expected to:
- (A) use relationships and objects to determine whether a number up to 40 is even or odd;
 - (B) use relationships to determine the number that is 10 or 100 more or less than a given number up to 1,200; and
 - (C) represent and solve addition and subtraction word problems where unknowns may be any one of the terms in the problem.
- (8) Geometry and measurement. The student applies mathematical process standards to analyze attributes of two- and three-dimensional geometric figures to develop generalizations about their properties. The student is expected to:
- (A) create two-dimensional shapes based on given attributes, including number of sides and vertices;
 - (B) identify attributes of a quadrilateral, a pentagon, and an octagon;
 - (C) classify and sort three-dimensional solids, including cones, cylinders, spheres, triangular and rectangular prisms, and cubes based on attributes using formal geometric language such as vertex, edge, and face;
 - (D) classify and sort polygons with 12 or fewer sides according to attributes, including identifying the number of sides and number of vertices;
 - (E) compose two-dimensional shapes and three-dimensional solids with given properties or attributes such as build a rectangle out of unit squares or build a rectangular prism out of unit cubes; and
 - (F) decompose two-dimensional shapes such as cutting out a square from a rectangle, dividing a shape in half, or partitioning a rectangle into identical triangles and identify the resulting geometric parts.
- (9) Geometry and measurement. The student applies mathematical process standards to select and use units to describe length, area, and time. The student is expected to:
- (A) find the length of objects using concrete models for standard units of length such as the edges of inch tiles or centimeter cubes;
 - (B) describe the inverse relationship between the size of the unit and the number of units needed to equal the length of an object such as the longer the unit, the fewer needed and the shorter the unit, the more needed;
 - (C) represent whole numbers as distances from any given location on a number line;
 - (D) determine the length of an object to the nearest half unit using rulers, yardsticks, meter sticks, or measuring tapes;

- (E) determine a solution to a problem involving length, including estimating lengths;
 - (F) use concrete models of square units to find the area of a rectangle by covering it with no gaps or overlaps, counting to find the total number of square units, and describing the measurement using a number and the unit such as 24 square units; and
 - (G) read and write time to the nearest five- and one-minute increments using analog and digital clocks and distinguish between a.m. and p.m.
- (10) Data analysis. The student applies mathematical process standards to organize data to make it useful for interpreting information and solving problems. The student is expected to:
- (A) explain that the length of a bar in a bar graph or the number of pictures in a pictograph represents the number of data points for a given category;
 - (B) organize a collection of data with up to four categories using pictographs and bar graphs with intervals of one or more;
 - (C) write and solve one-step word problems involving addition or subtraction using data represented within pictographs and bar graphs with intervals of one; and
 - (D) draw conclusions and make predictions from information in a graph.

§111.5. Grade 3, Beginning with School Year 2013-2014.

(a) Introduction.

- (1) The desire to achieve educational excellence is the driving force behind the Texas essential knowledge and skills for mathematics, guided by the college and career readiness standards. By embedding statistics, probability, and finance, while focusing on fluency and solid understanding, Texas will lead the way in mathematics education and prepare all Texas students for the challenges they will face in the 21st century.
- (2) The process standards describe ways in which students are expected to engage in the content. The placement of the process standards at the beginning of the knowledge and skills listed for each grade and course is intentional. The process standards weave the other knowledge and skills together so that students may be successful problem solvers and use mathematics efficiently and effectively in daily life. The process standards are integrated at every grade level and course. When possible, students will apply mathematics to problems arising in everyday life, society, and the workplace. Students will use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution. Students will select appropriate tools such as real objects, manipulatives, paper and pencil, and technology and techniques such as mental math, estimation, and number sense to solve problems. Students will effectively communicate mathematical ideas, reasoning, and their implications using multiple representations such as symbols, diagrams, graphs, and language. Students will use mathematical relationships to generate solutions and make connections and predictions. Students will analyze mathematical relationships to connect and communicate mathematical ideas. Students will display, explain, or justify mathematical ideas and arguments using precise mathematical language in written or oral communication.
- (3) For students to become fluent in mathematics, students must develop a robust sense of number. The National Research Council's report, "Adding It Up," defines procedural fluency as "skill in carrying out procedures flexibly, accurately, efficiently, and appropriately." As students develop procedural fluency, they must also realize that true problem solving may take time, effort, and perseverance. Students in Grade 3 are expected to perform their work without the use of calculators.
- (4) The primary focal areas in Grade 3 are place value, operations of whole numbers, and understanding fractional units. These focal areas are supported throughout the mathematical strands of number and operations, algebraic reasoning, geometry and measurement, and data analysis. In Grades 3-5, the number set is limited to positive rational numbers. In number and

operations, students will focus on applying place value, comparing and ordering whole numbers, connecting multiplication and division, and understanding and representing fractions as numbers and equivalent fractions. In algebraic reasoning, students will use multiple representations of problem situations, determine missing values in number sentences, and represent real-world relationships using number pairs in a table and verbal descriptions. In geometry and measurement, students will identify and classify two-dimensional figures according to common attributes, decompose composite figures formed by rectangles to determine area, determine the perimeter of polygons, solve problems involving time, and measure liquid volume (capacity) or weight. In data analysis, students will represent and interpret data.

(5) Statements that contain the word "including" reference content that must be mastered, while those containing the phrase "such as" are intended as possible illustrative examples.

(b) Knowledge and skills.

(1) Mathematical process standards. The student uses mathematical processes to acquire and demonstrate mathematical understanding. The student is expected to:

(A) apply mathematics to problems arising in everyday life, society, and the workplace;

(B) use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution;

(C) select tools, including real objects, manipulatives, paper and pencil, and technology as appropriate, and techniques, including mental math, estimation, and number sense as appropriate, to solve problems;

(D) communicate mathematical ideas, reasoning, and their implications using multiple representations, including symbols, diagrams, graphs, and language as appropriate;

(E) create and use representations to organize, record, and communicate mathematical ideas;

(F) analyze mathematical relationships to connect and communicate mathematical ideas; and

(G) display, explain, and justify mathematical ideas and arguments using precise mathematical language in written or oral communication.

(2) Number and operations. The student applies mathematical process standards to represent and compare whole numbers and understand relationships related to place value. The student is expected to:

(A) compose and decompose numbers up to 100,000 in more than one way as a sum of so many ten thousands, so many thousands, so many hundreds, so many tens, and so many ones using objects, pictorial models, and numbers, including expanded notation as appropriate;

(B) describe the mathematical relationships found in the base-10 place value system through the 100,000s place;

(C) represent a number on a number line as being between two consecutive multiples of 10; 100; 1,000; or 10,000 and use words such as "closer to," "is about," or "is nearly" to describe relative size of numbers in order to round whole numbers; and

(D) compare and order whole numbers up to 100,000 and represent comparisons using the symbols $>$, $<$, or $=$.

(3) Number and operations. The student applies mathematical process standards to represent and explain fractional units. The student is expected to:

(A) represent fractions greater than zero and less than or equal to one using concrete objects and pictorial models, including strip diagrams and number lines with denominators of 2, 3, 4, 6, and 8;

- (B) determine the corresponding fraction greater than zero and less than or equal to one with denominators of 2, 3, 4, 6, and 8 of a specified point on a number line;
 - (C) explain that the unit fraction $1/b$ represents the quantity formed by one part of a whole that has been partitioned into b equal parts where b is a non-zero whole number;
 - (D) compose and decompose a fraction a/b with a numerator greater than zero and less than or equal to b as a sum of parts $1/b$;
 - (E) solve problems involving partitioning an object or a set of objects among two or more recipients using pictorial representations of fractions with denominators of 2, 3, 4, 6, and 8 such as two children share five cookies;
 - (F) represent equivalent fractions with denominators of 2, 3, 4, 6, and 8 using a variety of objects and pictorial models, including number lines;
 - (G) explain that two fractions are equivalent if and only if they are both represented by the same point on the number line or represent the same portion of a same size whole for an area model; and
 - (H) compare two fractions having the same numerator or denominator in problems by reasoning about their sizes and justifying the conclusion using symbols, words, objects, and pictorial models such as comparing the size of pieces when sharing a candy bar equally among four people or equally among three people.
- (4) Number and operations. The student applies mathematical process standards to develop and use strategies and methods for whole number computations in order to solve problems with efficiency and accuracy. The student is expected to:
- (A) solve one-step and multi-step problems involving addition and subtraction within 1,000 using strategies based on place value, properties of operations, and the relationship between addition and subtraction with fluency;
 - (B) use strategies, including rounding to the nearest 10 or 100 and compatible numbers, to estimate solutions to addition and subtraction problems;
 - (C) determine the value of a collection of coins and bills;
 - (D) determine the total number of objects when equally-sized groups of objects are combined or arranged in arrays up to 10 by 10;
 - (E) represent multiplication facts by using a variety of approaches such as repeated addition, equal-sized groups, arrays, area models, equal jumps on a number line, and skip counting;
 - (F) quickly recall facts to multiply up to 10 by 10 and recall the corresponding division facts;
 - (G) use strategies and algorithms, including the standard algorithm, to multiply a two-digit number by a one-digit number. Strategies may include mental math, partial products, and the commutative, associative, and distributive properties;
 - (H) determine the number of objects in each group when a set of objects is partitioned into equal shares or a set of objects is shared equally;
 - (I) use divisibility rules to determine if a number is even or odd;
 - (J) determine a quotient using the relationship between multiplication and division such as the quotient of $40 \div 8$ can be found by determining what factor makes 40 when multiplied by 8; and
 - (K) solve one-step and multi-step problems involving multiplication and division within 100 using strategies based on objects; pictorial models, including arrays, area models, and equal groups; properties of operations; or recall of facts.

- (5) Algebraic reasoning. The student applies mathematical process standards to analyze and create patterns and relationships. The student is expected to:
- (A) represent and solve one- and two-step problems involving addition and subtraction of whole numbers to 1,000 using pictorial models such as strip diagrams and number lines and equations;
 - (B) represent and solve one- and two-step multiplication and division problems within 100 using arrays, strip diagrams, and equations;
 - (C) describe a multiplication expression as a comparison such as 3×24 represents 3 times as much as 24;
 - (D) determine the unknown whole number in a multiplication or division equation relating three whole numbers when the unknown is either a missing factor or product such as the value 4 makes $3 \times \square = 12$ a true equation; and
 - (E) represent real-world relationships using number pairs in a table and verbal descriptions such as 1 insect has 6 legs, 2 insects have 12 legs, and so forth.
- (6) Geometry and measurement. The student applies mathematical process standards to analyze attributes of two-dimensional geometric figures to develop generalizations about their properties. The student is expected to:
- (A) classify and sort two- and three-dimensional solids, including cones, cylinders, spheres, triangular and rectangular prisms, and cubes, based on attributes using formal geometric language such as vertex, edge, and face;
 - (B) determine the area of rectangles with whole number side lengths in problems using multiplication related to the number of rows times the number of unit squares in each row;
 - (C) decompose composite figures formed by rectangles into non-overlapping rectangles to determine the area of the original figure using the additive property of area; and
 - (D) decompose two congruent two-dimensional figures into parts with equal areas and express the area of each part as a unit fraction of the whole and recognize that equal shares of identical wholes need not have the same shape.
- (7) Geometry and measurement. The student applies mathematical process standards to select appropriate units, strategies, and tools to solve problems involving customary measurement. The student is expected to:
- (A) represent fractions of halves, fourths, and eighths as distances from zero on a number line;
 - (B) determine the perimeter of a polygon or a missing length when given perimeter and remaining side lengths in problems;
 - (C) determine the solutions to problems involving addition and subtraction of time intervals in minutes using pictorial models or tools such as a 15-minute event plus a 30-minute event equals 45 minutes;
 - (D) determine when it is appropriate to use measurements of liquid volume (capacity) or weight; and
 - (E) determine liquid volume (capacity) or weight using appropriate units and tools.
- (8) Data analysis. The student applies mathematical process standards to solve problems by collecting, organizing, displaying, and interpreting data. The student is expected to:
- (A) summarize a data set with multiple categories using a frequency table, dot plot, pictograph, or bar graph with scaled intervals; and

- (B) solve one- and two-step problems using categorical data represented with a frequency table, dot plot, pictograph, or bar graph with scaled intervals.

§111.6. Grade 4, Beginning with School Year 2013-2014.

(a) Introduction.

- (1) The desire to achieve educational excellence is the driving force behind the Texas essential knowledge and skills for mathematics, guided by the college and career readiness standards. By embedding statistics, probability, and finance, while focusing on fluency and solid understanding, Texas will lead the way in mathematics education and prepare all Texas students for the challenges they will face in the 21st century.
- (2) The process standards describe ways in which students are expected to engage in the content. The placement of the process standards at the beginning of the knowledge and skills listed for each grade and course is intentional. The process standards weave the other knowledge and skills together so that students may be successful problem solvers and use mathematics efficiently and effectively in daily life. The process standards are integrated at every grade level and course. When possible, students will apply mathematics to problems arising in everyday life, society, and the workplace. Students will use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution. Students will select appropriate tools such as real objects, manipulatives, paper and pencil, and technology and techniques such as mental math, estimation, and number sense to solve problems. Students will effectively communicate mathematical ideas, reasoning, and their implications using multiple representations such as symbols, diagrams, graphs, and language. Students will use mathematical relationships to generate solutions and make connections and predictions. Students will analyze mathematical relationships to connect and communicate mathematical ideas. Students will display, explain, or justify mathematical ideas and arguments using precise mathematical language in written or oral communication.
- (3) For students to become fluent in mathematics, students must develop a robust sense of number. The National Research Council's report, "Adding It Up," defines procedural fluency as "skill in carrying out procedures flexibly, accurately, efficiently, and appropriately." As students develop procedural fluency, they must also realize that true problem solving may take time, effort, and perseverance. Students in Grade 4 are expected to perform their work without the use of calculators.
- (4) The primary focal areas in Grade 4 are use of operations, fractions, and decimals and describing and analyzing geometry and measurement. These focal areas are supported throughout the mathematical strands of number and operations, algebraic reasoning, geometry and measurement, and data analysis. In Grades 3-5, the number set is limited to positive rational numbers. In number and operations, students will apply place value and represent points on a number line that correspond to a given fraction or terminating decimal. In algebraic reasoning, students will represent and solve multi-step problems involving the four operations with whole numbers with expressions and equations and generate and analyze patterns. In geometry and measurement, students will classify two-dimensional figures, measure angles, and convert units of measure. In data analysis, students will represent and interpret data.
- (5) Statements that contain the word "including" reference content that must be mastered, while those containing the phrase "such as" are intended as possible illustrative examples.

(b) Knowledge and skills.

- (1) Mathematical process standards. The student uses mathematical processes to acquire and demonstrate mathematical understanding. The student is expected to:
- (A) apply mathematics to problems arising in everyday life, society, and the workplace;

- (B) use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution;
 - (C) select tools, including real objects, manipulatives, paper and pencil, and technology as appropriate, and techniques, including mental math, estimation, and number sense as appropriate, to solve problems;
 - (D) communicate mathematical ideas, reasoning, and their implications using multiple representations, including symbols, diagrams, graphs, and language as appropriate;
 - (E) create and use representations to organize, record, and communicate mathematical ideas;
 - (F) analyze mathematical relationships to connect and communicate mathematical ideas; and
 - (G) display, explain, and justify mathematical ideas and arguments using precise mathematical language in written or oral communication.
- (2) Number and operations. The student applies mathematical process standards to represent, compare, and order whole numbers and decimals and understand relationships related to place value. The student is expected to:
- (A) interpret the value of each place-value position as 10 times the position to the right and as one-tenth of the value of the place to its left;
 - (B) represent the value of the digit in whole numbers through 1,000,000,000 and decimals to the hundredths using expanded notation and numerals such as in the number 3.94, the 3 in the ones place is 3; the 9 in the tenths place is 0.9; and the 4 in the hundredths place is 0.04; and 3.94 is the sum of 3 ones, 9 tenths, and 4 hundredths;
 - (C) compare and order whole numbers to 1,000,000,000 and represent comparisons using the symbols $>$, $<$, or $=$;
 - (D) round whole numbers to a given place value through the 100,000s place;
 - (E) represent decimals, including tenths and hundredths, using concrete and visual models and money;
 - (F) compare and order decimals using concrete and visual models to the hundredths;
 - (G) relate decimals to fractions that name tenths and hundredths; and
 - (H) determine the corresponding decimal to the tenths or hundredths place of a specified point on a number line.
- (3) Number and operations. The student applies mathematical process standards to represent and generate fractions to solve problems. The student is expected to:
- (A) represent a fraction a/b as a sum of fractions $1/b$, where a and b are whole numbers and $b > 0$, including when $a > b$;
 - (B) decompose a fraction in more than one way into a sum of fractions with the same denominator using concrete and pictorial models and recording results with symbolic representations such as $7/8 = 5/8 + 2/8$; $7/8 = 3/8 + 4/8$; $2\ 7/8 = 1 + 1 + 7/8$; $2\ 7/8 = 8/8 + 8/8 + 7/8$;
 - (C) determine if two given fractions are equivalent using a variety of methods, including multiplying by a fraction equivalent to one or simplifying a fraction to lowest terms;
 - (D) generate equivalent fractions to create equal numerators or equal denominators to compare two fractions with unequal numerators and unequal denominators and represent the comparison of two fractions using the symbols $>$, $<$, or $=$;

- (E) represent and solve addition and subtraction of fractions with equal denominators and referring to the same whole using objects and pictorial models that build to the number line such as strip diagrams and properties of operations;
 - (F) estimate the reasonableness of sums and differences using benchmark fractions 0, 1/4, 1/2, 3/4, and 1, referring to the same whole;
 - (G) represent fractions and decimals to the tenths or hundredths as distances from zero on a number line; and
 - (H) determine fractional and decimal quantities as being close to 0, 1/2, and 1.
- (4) Number and operations. The student applies mathematical process standards to develop and use strategies and methods for whole number computations and decimal sums and differences in order to solve problems with efficiency and accuracy. The student is expected to:
- (A) add and subtract whole numbers and decimals to the hundredths place using a variety of methods, including pictorial models, the inverse relationship between operations, concepts of place value, and efficient algorithms;
 - (B) determine products of a number and 10 or 100 using properties of operations and place value understandings;
 - (C) represent the product of 2 two-digit numbers using arrays, area models, or equations, including perfect squares through 15 by 15;
 - (D) use strategies and algorithms, including the standard algorithm, to multiply up to a four-digit number by a one-digit number and to multiply a two-digit number by a two-digit number. Strategies may include mental math, partial products, and the commutative, associative, and distributive properties;
 - (E) represent the quotient of up to a four-digit whole number divided by a one-digit whole number using arrays, area models, or equations;
 - (F) use strategies and algorithms, including the standard algorithm, to divide up to a four-digit dividend by a one-digit divisor;
 - (G) use strategies, including rounding to the nearest 10, 100, or 1,000 and compatible numbers, to estimate solutions; and
 - (H) solve one- and two-step problems involving multiplication and division with fluency, including interpreting remainders.
- (5) Algebraic reasoning. The student applies mathematical process standards to develop concepts of expressions and equations. The student is expected to:
- (A) represent multi-step problems involving the four operations with whole numbers using strip diagrams and equations with a letter standing for the unknown quantity;
 - (B) represent problems using an input-output table and numerical expressions to generate a number pattern that follows a given rule such as given the rule "Add 3" and the starting number 1, use the expressions $1 + 3$, $2 + 3$, $3 + 3$, and so forth to generate a table to represent the relationship of the values in the resulting sequence and their position in the sequence;
 - (C) use models to determine the formulas for the perimeter of a rectangle ($l + w + l + w$ or $2l + 2w$), including the special form for perimeter of a square ($4s$) and the area of a rectangle ($l \times w$); and
 - (D) solve problems related to perimeter and area of rectangles where dimensions are whole numbers.

- (6) Geometry and measurement. The student applies mathematical process standards to analyze geometric attributes in order to develop generalizations about their properties. The student is expected to:
- (A) identify points, lines, line segments, rays, angles, and perpendicular and parallel lines;
 - (B) identify and draw one or more lines of symmetry, if they exist, for a two-dimensional figure;
 - (C) apply knowledge of right angles to identify acute, right, and obtuse triangles;
 - (D) use attributes to recognize rhombuses, parallelograms, trapezoids, rectangles, and squares as examples of quadrilaterals and draw examples of quadrilaterals that do not belong to any of these subcategories; and
 - (E) classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines or the presence or absence of angles of a specified size.
- (7) Geometry and measurement. The student applies mathematical process standards to solve problems involving angles less than or equal to 180 degrees. The student is expected to:
- (A) illustrate the measure of an angle as the part of a circle whose center is at the vertex of the angle that is "cut out" by the rays of the angle. Angle measures are limited to whole numbers;
 - (B) illustrate degrees as the units used to measure an angle, where $1/360$ of any circle is one degree and an angle that "cuts" $n/360$ out of any circle whose center is at the angle's vertex has a measure of n degrees. Angle measures are limited to whole numbers;
 - (C) determine the approximate measures of angles in degrees to the nearest whole number using a protractor;
 - (D) draw an angle with a given measure; and
 - (E) decompose angles such as complementary and supplementary angles into two non-overlapping angles to determine the measure of an unknown angle.
- (8) Geometry and measurement. The student applies mathematical process standards to select appropriate customary and metric units, strategies, and tools to solve problems involving measurement. The student is expected to:
- (A) identify relative sizes of measurement units within the customary and metric systems;
 - (B) convert measurements within the same measurement system, customary or metric, from a smaller unit into a larger unit or a larger unit into a smaller unit when given other equivalent measures represented in a table; and
 - (C) solve problems that deal with measurements of length, intervals of time, liquid volumes, masses, and money using addition, subtraction, multiplication, or division as appropriate.
- (9) Data analysis. The student applies mathematical process standards to solve problems by collecting, organizing, displaying, and interpreting data. The student is expected to:
- (A) represent data on a frequency table, dot plot, or stem-and-leaf plot marked with whole numbers and fractions; and
 - (B) solve one- and two-step problems using data in whole number, decimal, and fraction form in a frequency table, dot plot, or stem-and-leaf plot.

§111.7. Grade 5, Beginning with School Year 2013-2014.

(a) Introduction.

- (1) The desire to achieve educational excellence is the driving force behind the Texas essential knowledge and skills for mathematics, guided by the college and career readiness standards. By

embedding statistics, probability, and finance, while focusing on fluency and solid understanding. Texas will lead the way in mathematics education and prepare all Texas students for the challenges they will face in the 21st century.

- (2) The process standards describe ways in which students are expected to engage in the content. The placement of the process standards at the beginning of the knowledge and skills listed for each grade and course is intentional. The process standards weave the other knowledge and skills together so that students may be successful problem solvers and use mathematics efficiently and effectively in daily life. The process standards are integrated at every grade level and course. When possible, students will apply mathematics to problems arising in everyday life, society, and the workplace. Students will use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution. Students will select appropriate tools such as real objects, manipulatives, paper and pencil, and technology and techniques such as mental math, estimation, and number sense to solve problems. Students will effectively communicate mathematical ideas, reasoning, and their implications using multiple representations such as symbols, diagrams, graphs, and language. Students will use mathematical relationships to generate solutions and make connections and predictions. Students will analyze mathematical relationships to connect and communicate mathematical ideas. Students will display, explain, or justify mathematical ideas and arguments using precise mathematical language in written or oral communication.
 - (3) For students to become fluent in mathematics, students must develop a robust sense of number. The National Research Council's report, "Adding It Up," defines procedural fluency as "skill in carrying out procedures flexibly, accurately, efficiently, and appropriately." As students develop procedural fluency, they must also realize that true problem solving may take time, effort, and perseverance. Students in Grade 5 are expected to perform their work without the use of calculators.
 - (4) The primary focal areas in Grade 5 are solving problems involving all four operations with positive rational numbers, determining and generating formulas and solutions to expressions, and extending measurement to area and volume. These focal areas are supported throughout the mathematical strands of number and operations, algebraic reasoning, geometry and measurement, and data analysis. In Grades 3-5, the number set is limited to positive rational numbers. In number and operations, students will apply place value and identify part-to-whole relationships and equivalence. In algebraic reasoning, students will represent and solve problems with expressions and equations, build foundations of functions through patterning, identify prime and composite numbers, and use the order of operations. In geometry and measurement, students will classify two-dimensional figures, connect geometric attributes to the measures of three-dimensional figures, use units of measure, and represent location using a coordinate plane. In data analysis, students will represent and interpret data.
 - (5) Statements that contain the word "including" reference content that must be mastered, while those containing the phrase "such as" are intended as possible illustrative examples.
- (b) Knowledge and skills.
- (1) Mathematical process standards. The student uses mathematical processes to acquire and demonstrate mathematical understanding. The student is expected to:
 - (A) apply mathematics to problems arising in everyday life, society, and the workplace;
 - (B) use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution;
 - (C) select tools, including real objects, manipulatives, paper and pencil, and technology as appropriate, and techniques, including mental math, estimation, and number sense as appropriate, to solve problems;

- (D) communicate mathematical ideas, reasoning, and their implications using multiple representations, including symbols, diagrams, graphs, and language as appropriate;
 - (E) create and use representations to organize, record, and communicate mathematical ideas;
 - (F) analyze mathematical relationships to connect and communicate mathematical ideas; and
 - (G) display, explain, and justify mathematical ideas and arguments using precise mathematical language in written or oral communication.
- (2) Number and operations. The student applies mathematical process standards to represent, compare, and order positive rational numbers and understand relationships as related to place value. The student is expected to:
- (A) interpret the value of each place-value position as one-tenth of the value of the place to its left or as 10 times the value of the place to its right;
 - (B) represent the value of the digit in decimals through the thousandths using expanded notation and numerals;
 - (C) compare and order two decimals to thousandths and represent comparisons using the symbols $>$, $<$, or $=$; and
 - (D) round decimals to tenths or hundredths.
- (3) Number and operations. The student applies mathematical process standards to develop and use strategies and methods for positive rational number computations in order to solve problems with efficiency and accuracy. The student is expected to:
- (A) estimate to determine solutions to mathematical and real-world problems involving addition, subtraction, multiplication, or division;
 - (B) use strategies and algorithms, including the standard algorithm, to multiply a three-digit number by a two-digit number with fluency;
 - (C) use strategies and algorithms, including the standard algorithm, to solve for quotients of up to a four-digit dividend and a two-digit divisor with fluency;
 - (D) represent multiplication of decimals with products to the hundredths using objects and pictorial models, including area models;
 - (E) solve for products of decimals to the hundredths, including situations involving money, using strategies based on place-value understandings, properties of operations, and the relationship to the multiplication of whole numbers;
 - (F) represent quotients to the hundredths, up to four-digit dividends and two-digit whole number divisors, using objects and pictorial models, including area models;
 - (G) solve for quotients to the hundredths, up to four-digit dividends and two-digit whole number divisors, using strategies and algorithms, including the standard algorithm;
 - (H) represent and solve addition and subtraction of fractions with unequal denominators referring to the same whole using objects and pictorial models such as strip diagrams and properties of operations;
 - (I) represent and solve multiplication of a whole number and a fraction that refers to the same whole using objects and pictorial models, including area models; and
 - (J) represent division of a unit fraction by a whole number and the division of a whole number by a unit fraction such as $1/3 \div 7$ and $7 \div 1/3$ using objects and pictorial models, including area models.
- (4) Algebraic reasoning. The student applies mathematical process standards to develop concepts of expressions and equations. The student is expected to:
- (A) identify prime and composite numbers using patterns in factor pairs;

- (B) represent and solve multi-step problems involving the four operations with whole numbers using equations with a letter standing for the unknown quantity;
 - (C) generate a numerical pattern when given a rule in the form $y = ax$ or $y = x + a$ and graph;
 - (D) recognize the difference between additive and multiplicative numerical patterns given in a table or graph;
 - (E) describe the meaning of parentheses and brackets in a numeric expression such as $4(14 + 5)$ is 4 times as large as $(14 + 5)$;
 - (F) simplify numerical expressions, including up to two levels of grouping excluding exponents such as $(3 + 7) / (5 - 3)$;
 - (G) use concrete objects and pictorial models to develop the formulas for the volume of a rectangular prism, including the special form for a cube ($V = l \times w \times h$, $V = s \times s \times s$, and $V = Bh$); and
 - (H) represent and solve problems related to perimeter and/or area such as rectangles and composite figures formed by rectangles and related to volume such as rectangular prisms.
- (5) Geometry and measurement. The student applies mathematical process standards to classify two-dimensional figures by attributes and properties. The student is expected to classify two-dimensional figures in a hierarchy of sets and subsets using graphic organizers based on their attributes and properties such as all rectangles have the property that opposite sides are parallel; therefore, every rectangle is a parallelogram.
- (6) Geometry and measurement. The student applies mathematical process standards to understand, recognize, and quantify volume. The student is expected to:
- (A) recognize a cube with side length of one unit as a unit cube having one cubic unit of volume and the volume of a three-dimensional figure as the number of unit cubes (n cubic units) needed to fill it with no gaps or overlaps if possible; and
 - (B) determine the volume of a rectangular prism with whole number side lengths in problems related to the number of layers times the number of unit cubes in the area of the base.
- (7) Geometry and measurement. The student applies mathematical process standards to select appropriate units, strategies, and tools to solve problems involving measurement. The student is expected to solve problems by calculating conversions within a measurement system, customary or metric.
- (8) Geometry and measurement. The student applies mathematical process standards to identify locations on a coordinate plane. The student is expected to:
- (A) describe the key attributes of the coordinate plane and the process for graphing ordered pairs of numbers in the first quadrant; and
 - (B) graph ordered pairs of numbers arising from mathematical and real-world problems in the first quadrant of the coordinate plane, including those generated by number patterns or found in an input-output table.
- (9) Data analysis. The student applies mathematical process standards to solve problems by collecting, organizing, displaying, and interpreting data. The student is expected to:
- (A) represent categorical data with bar graphs or frequency tables and numerical data, including data sets of measurements in fractions or decimals, with dot plots or stem-and-leaf plots;
 - (B) represent discrete paired data on a scatterplot; and
 - (C) solve one- and two-step problems using data from a frequency table, dot plot, bar graph, stem-and-leaf plot, or scatterplot.

Subchapter B. Middle School

§111.21. Implementation of Texas Essential Knowledge and Skills for Mathematics, Middle School, Beginning with School Year 2013-2014.

The provisions of §§111.22-111.24 of this subchapter shall be implemented by school districts beginning with the 2013-2014 school year. This implementation date shall supersede any other implementation dates found in this subchapter.

§111.22. Grade 6, Beginning with School Year 2013-2014.

(a) Introduction.

- (1) The desire to achieve educational excellence is the driving force behind the Texas essential knowledge and skills for mathematics, guided by the college and career readiness standards. By embedding statistics, probability, and finance, while focusing on fluency and solid understanding, Texas will lead the way in mathematics education and prepare all Texas students for the challenges they will face in the 21st century.
- (2) The process standards describe ways in which students are expected to engage in the content. The placement of the process standards at the beginning of the knowledge and skills listed for each grade and course is intentional. The process standards weave the other knowledge and skills together so that students may be successful problem solvers and use mathematics efficiently and effectively in daily life. The process standards are integrated at every grade level and course. When possible, students will apply mathematics to problems arising in everyday life, society, and the workplace. Students will use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution. Students will select appropriate tools such as real objects, manipulatives, paper and pencil, and technology and techniques such as mental math, estimation, and number sense to solve problems. Students will effectively communicate mathematical ideas, reasoning, and their implications using multiple representations such as symbols, diagrams, graphs, and language. Students will use mathematical relationships to generate solutions and make connections and predictions. Students will analyze mathematical relationships to connect and communicate mathematical ideas. Students will display, explain, or justify mathematical ideas and arguments using precise mathematical language in written or oral communication.
- (3) The primary focal areas in Grade 6 are number and operations; proportionality; expressions, equations, and relationships; and measurement and data. Students use concepts, algorithms, and properties of rational numbers to explore mathematical relationships and to describe increasingly complex situations. Students use concepts of proportionality to explore, develop, and communicate mathematical relationships. Students use algebraic thinking to describe how a change in one quantity in a relationship results in a change in the other. Students connect verbal, numeric, graphic, and symbolic representations of relationships, including equations and inequalities. Students use geometric properties and relationships, as well as spatial reasoning, to model and analyze situations and solve problems. Students communicate information about geometric figures or situations by quantifying attributes, generalize procedures from measurement experiences, and use the procedures to solve problems. Students use appropriate statistics, representations of data, and reasoning to draw conclusions, evaluate arguments, and make recommendations. While the use of all types of technology is important, the emphasis on algebra readiness skills necessitates the implementation of graphing technology.
- (4) Statements that contain the word "including" reference content that must be mastered, while those containing the phrase "such as" are intended as possible illustrative examples.

(b) Knowledge and skills.

- (1) Mathematical process standards. The student uses mathematical processes to acquire and demonstrate mathematical understanding. The student is expected to:
- (A) apply mathematics to problems arising in everyday life, society, and the workplace;
 - (B) use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution;
 - (C) select tools, including real objects, manipulatives, paper and pencil, and technology as appropriate, and techniques, including mental math, estimation, and number sense as appropriate, to solve problems;
 - (D) communicate mathematical ideas, reasoning, and their implications using multiple representations, including symbols, diagrams, graphs, and language as appropriate;
 - (E) create and use representations to organize, record, and communicate mathematical ideas;
 - (F) analyze mathematical relationships to connect and communicate mathematical ideas; and
 - (G) display, explain, and justify mathematical ideas and arguments using precise mathematical language in written or oral communication.
- (2) Number and operations. The student applies mathematical process standards to represent and use rational numbers in a variety of forms. The student is expected to:
- (A) classify whole numbers, integers, and rational numbers using a visual representation such as a Venn diagram to describe relationships between sets of numbers;
 - (B) identify a number, its opposite, and its absolute value;
 - (C) locate, compare, and order integers using a number line;
 - (D) locate, compare, and order rational numbers using a number line;
 - (E) order a set of rational numbers arising from mathematical and real-world contexts; and
 - (F) extend representations for division to include fraction notation such as a/b represents the same number as $a \div b$ where $b \neq 0$.
- (3) Number and operations. The student applies mathematical process standards to represent addition, subtraction, multiplication, and division while solving problems and justifying solutions. The student is expected to:
- (A) represent division of decimals to the hundredths place using models and connect to an algorithm;
 - (B) use an area model to represent fraction multiplication and decimal multiplication;
 - (C) recognize that dividing by a rational number and multiplying by its reciprocal result in equivalent values;
 - (D) determine whether a quantity is increased or decreased when multiplied by a fraction, including values greater than or less than one, with and without computation;
 - (E) represent integer operations with concrete models and connect the actions to algorithms;
 - (F) use prior knowledge of all four operations, including whole numbers and positive decimals, fractions, and mixed numbers not having fractions and decimals, within the same problem;
 - (G) add, subtract, multiply, and divide integers; and
 - (H) add and subtract positive rational numbers fluently.
- (4) Proportionality. The student applies mathematical process standards to develop an understanding of proportional relationships in problem situations. The student is expected to:

- (A) compare two rules verbally, numerically, graphically, and symbolically in the form of $y = ax$ or $y = x + a$ in order to differentiate between additive and multiplicative relationships;
 - (B) apply qualitative and quantitative reasoning to solve prediction and comparison of real-world problems involving ratios and rates;
 - (C) give examples of ratios as multiplicative comparisons of two quantities describing the same attribute;
 - (D) give examples of rates as the comparison by division of two quantities having different attributes, including rates as quotients;
 - (E) represent ratios and percents with concrete models, fractions, and decimals;
 - (F) represent benchmark fractions and percents such as 1%, 10%, 25%, $33\frac{1}{3}\%$, and multiples of these values using 10 by 10 grids, strip diagrams, number lines, and numbers; and
 - (G) generate equivalent forms of fractions, decimals, and percents using real-world problems, including problems that involve money.
- (5) Proportionality. The student applies mathematical process standards to solve problems involving proportional relationships. The student is expected to:
- (A) represent mathematical and real-world problems involving ratios and rates using scale factors, tables, graphs, and proportions;
 - (B) solve real-world problems finding the part given the whole and the percent and finding the percent given the part and the whole using concrete and pictorial models; and
 - (C) use equivalent fractions, decimals, and percents to show equal parts of the same whole.
- (6) Expressions, equations, and relationships. The student applies mathematical process standards to use multiple representations to describe algebraic relationships. The student is expected to:
- (A) identify independent and dependent quantities from tables and graphs;
 - (B) write an equation that represents the relationship between independent and dependent quantities from a table; and
 - (C) represent a given situation using verbal descriptions, tables, graphs, and equations in the form $y = kx$ or $y = x + b$.
- (7) Expressions, equations, and relationships. The student applies mathematical process standards to develop concepts of expressions and equations. The student is expected to:
- (A) generate equivalent numerical expressions using order of operations, including positive exponents and prime factorization;
 - (B) distinguish between expressions and equations verbally, numerically, and algebraically;
 - (C) determine if two expressions are equivalent using concrete models, pictorial models, and algebraic representations; and
 - (D) generate equivalent expressions using the properties of operations such as the inverse, identity, commutative, associative, and distributive properties.
- (8) Expressions, equations, and relationships. The student applies mathematical process standards to use geometry to represent relationships and solve problems. The student is expected to:
- (A) extend previous knowledge of triangles and their properties to include the sum of angles of a triangle, the relationship between the lengths of sides and measures of angles in a triangle, and determining when three lengths form a triangle;
 - (B) model area formulas for parallelograms, trapezoids, and triangles by decomposing and rearranging parts of these shapes;

- (C) write equations that represent problems related to the area of rectangles, parallelograms, trapezoids, and triangles and volume of right rectangular prisms where dimensions are positive rational numbers; and
 - (D) determine solutions for problems involving the area of rectangles, parallelograms, trapezoids, and triangles and volume of right rectangular prisms where dimensions are positive rational numbers.
- (9) Expressions, equations, and relationships. The student applies mathematical process standards to use equations and inequalities to represent situations. The student is expected to:
- (A) write one-variable, one-step equations and inequalities to represent constraints or conditions within problems;
 - (B) represent solutions for one-variable, one-step equations and inequalities on number lines; and
 - (C) write corresponding real-world problems given one-variable, one-step equations or inequalities.
- (10) Expressions, equations, and relationships. The student applies mathematical process standards to use equations and inequalities to solve problems. The student is expected to:
- (A) model and solve one-variable, one-step equations and inequalities that represent problems, including geometric concepts such as complementary and supplementary angles; and
 - (B) determine if the given value(s) make(s) one-variable, one-step equations or inequalities true.
- (11) Measurement and data. The student applies mathematical process standards to use coordinate geometry to identify locations on a plane. The student is expected to graph points in all four quadrants using ordered pairs of rational numbers.
- (12) Measurement and data. The student applies mathematical process standards to use numerical or graphical representations to analyze problems. The student is expected to:
- (A) represent numeric data graphically, including dot plots, stem-and-leaf plots, histograms, and box plots;
 - (B) use the graphical representation of numeric data to describe the center, spread, and shape of the data distribution;
 - (C) summarize numeric data with numerical summaries, including the mean and median (measures of center) and the range and interquartile range (IQR) (measures of spread), and use these summaries to describe the center, spread, and shape of the data distribution; and
 - (D) summarize categorical data with numerical and graphical summaries, including the mode, the percent of values in each category (relative frequency table), and the percent bar graph, and use these summaries to describe the data distribution.
- (13) Measurement and data. The student applies mathematical process standards to use numerical or graphical representations to solve problems. The student is expected to:
- (A) interpret numeric data summarized in dot plots, stem-and-leaf plots, histograms, and box plots; and
 - (B) distinguish between situations that yield data with and without variability such as the question "How tall am I?" which would be answered with a single height versus the question "How tall are the students in my class?" which would be answered based on heights that vary.

§111.23. Grade 7, Beginning with School Year 2013-2014.

(a) Introduction.

- (1) The desire to achieve educational excellence is the driving force behind the Texas essential knowledge and skills for mathematics, guided by the college and career readiness standards. By embedding statistics, probability, and finance, while focusing on fluency and solid understanding, Texas will lead the way in mathematics education and prepare all Texas students for the challenges they will face in the 21st century.
- (2) The process standards describe ways in which students are expected to engage in the content. The placement of the process standards at the beginning of the knowledge and skills listed for each grade and course is intentional. The process standards weave the other knowledge and skills together so that students may be successful problem solvers and use mathematics efficiently and effectively in daily life. The process standards are integrated at every grade level and course. When possible, students will apply mathematics to problems arising in everyday life, society, and the workplace. Students will use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution. Students will select appropriate tools such as real objects, manipulatives, paper and pencil, and technology and techniques such as mental math, estimation, and number sense to solve problems. Students will effectively communicate mathematical ideas, reasoning, and their implications using multiple representations such as symbols, diagrams, graphs, and language. Students will use mathematical relationships to generate solutions and make connections and predictions. Students will analyze mathematical relationships to connect and communicate mathematical ideas. Students will display, explain, or justify mathematical ideas and arguments using precise mathematical language in written or oral communication.
- (3) The primary focal areas in Grade 7 are number and operations; proportionality; expressions, equations, and relationships; and measurement and data. Students use concepts, algorithms, and properties of rational numbers to explore mathematical relationships and to describe increasingly complex situations. Students use concepts of proportionality to explore, develop, and communicate mathematical relationships, including number, geometry and measurement, and statistics and probability. Students use algebraic thinking to describe how a change in one quantity in a relationship results in a change in the other. Students connect verbal, numeric, graphic, and symbolic representations of relationships, including equations and inequalities. Students use geometric properties and relationships, as well as spatial reasoning, to model and analyze situations and solve problems. Students communicate information about geometric figures or situations by quantifying attributes, generalize procedures from measurement experiences, and use the procedures to solve problems. Students use appropriate statistics, representations of data, and reasoning to draw conclusions, evaluate arguments, and make recommendations. While the use of all types of technology is important, the emphasis on algebra readiness skills necessitates the implementation of graphing technology.
- (4) Statements that contain the word "including" reference content that must be mastered, while those containing the phrase "such as" are intended as possible illustrative examples.

(b) Knowledge and skills.

- (1) Mathematical process standards. The student uses mathematical processes to acquire and demonstrate mathematical understanding. The student is expected to:

 - (A) apply mathematics to problems arising in everyday life, society, and the workplace;
 - (B) use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution;
 - (C) select tools, including real objects, manipulatives, paper and pencil, and technology as appropriate, and techniques, including mental math, estimation, and number sense as appropriate, to solve problems;

- (D) communicate mathematical ideas, reasoning, and their implications using multiple representations, including symbols, diagrams, graphs, and language as appropriate;
 - (E) create and use representations to organize, record, and communicate mathematical ideas;
 - (F) analyze mathematical relationships to connect and communicate mathematical ideas; and
 - (G) display, explain, and justify mathematical ideas and arguments using precise mathematical language in written or oral communication.
- (2) Number and operations. The student applies mathematical process standards to represent and use rational numbers in a variety of forms. The student is expected to extend previous knowledge of sets and subsets using a visual representation such as a Venn diagram to describe relationships between sets of rational numbers.
- (3) Number and operations. The student applies mathematical process standards to add, subtract, multiply, and divide while solving problems and justifying solutions. The student is expected to:
- (A) add, subtract, multiply, and divide integers fluently; and
 - (B) apply and extend previous understandings of operations to solve problems using addition, subtraction, multiplication, and division of rational numbers.
- (4) Proportionality. The student applies mathematical process standards to represent and solve problems involving proportional relationships. The student is expected to:
- (A) represent constant rates of change in mathematical and real-world problems given pictorial, tabular, verbal, numeric, graphical, and algebraic representations, including $d = rt$;
 - (B) calculate unit rates from rates in mathematical and real-world problems;
 - (C) determine the constant of proportionality ($k = y/x$) within mathematical and real-world problems;
 - (D) solve real-world problems to find the whole given a part and the percent, to find the part given the whole and the percent, and to find the percent given the part and the whole;
 - (E) solve problems involving ratios, rates, and percents, including multi-step problems involving percent increase and percent decrease, and financial literacy problems such as tax, tip, discount, simple interest, and commission;
 - (F) convert units within a measurement system, including the use of proportions and unit rates; and
 - (G) convert between measurement systems, including the use of proportions and the use of unit rates.
- (5) Proportionality. The student applies mathematical process standards to use geometry to describe or solve problems involving proportional relationships. The student is expected to:
- (A) generalize the critical attributes of similarity, including ratios within and between similar shapes;
 - (B) describe π as the ratio of the circumference of a circle to its diameter; and
 - (C) solve mathematical and real-world problems involving similar shape and scale drawings.
- (6) Proportionality. The student applies mathematical process standards to develop concepts of probability for simple and compound events. The student is expected to:
- (A) represent sample spaces for simple and compound events using lists and tree diagrams; and
 - (B) select and use different simulations to represent simple and compound events with and without technology.

- (7) Proportionality. The student applies mathematical process standards to make predictions and determine solutions for simple and compound events. The student is expected to:
- (A) make predictions and determine solutions using experimental data for simple and compound events; and
 - (B) make predictions and determine solutions using theoretical probability for simple and compound events.
- (8) Proportionality. The student applies mathematical process standards to find solutions in probability and statistics. The student is expected to:
- (A) find the probabilities of a simple event and its complement and describe the relationship between the two;
 - (B) use data from a random sample to make inferences about a population;
 - (C) solve problems using data represented in bar graphs, dot plots, and circle graphs, including part-to-whole and part-to-part comparisons and equivalents;
 - (D) solve problems using qualitative and quantitative predictions and comparisons from simple experiments; and
 - (E) determine experimental and theoretical probabilities related to simple and compound events using data and sample spaces.
- (9) Expressions, equations, and relationships. The student applies mathematical process standards to represent linear relationships using multiple representations. The student is expected to represent linear relationships using verbal descriptions, tables, graphs, and equations that simplify to the form $y = mx + b$.
- (10) Expressions, equations, and relationships. The student applies mathematical process standards to develop geometric relationships with volume. The student is expected to:
- (A) model the relationship between the volume of a rectangular prism and a rectangular pyramid having both congruent bases and heights and connect that relationship to the formulas;
 - (B) explain verbally and symbolically the relationship between the volume of a triangular prism and a triangular pyramid having both congruent bases and heights and connect that relationship to the formulas; and
 - (C) use models to determine the approximate formulas for the circumference and area of a circle and connect the models to the actual formulas.
- (11) Expressions, equations, and relationships. The student applies mathematical process standards to solve geometric problems. The student is expected to:
- (A) solve problems involving the volume of rectangular prisms, triangular prisms, rectangular pyramids, and triangular pyramids;
 - (B) determine the circumference and area of circles;
 - (C) determine the area of composite figures containing any combination of rectangles, squares, parallelograms, trapezoids, triangles, semicircles, and quarter circles; and
 - (D) solve problems involving the lateral and total surface area of a rectangular prism, rectangular pyramid, triangular prism, and triangular pyramid by determining the area of the shape's net.
- (12) Expressions, equations, and relationships. The student applies mathematical process standards to use one-variable equations and inequalities to represent situations. The student is expected to:
- (A) write one-variable, two-step equations and inequalities to represent constraints or conditions within problems;

- (B) represent solutions for one-variable, two-step equations and inequalities on number lines; and
 - (C) write a corresponding real-world problem given a one-variable, two-step equation or inequality.
- (13) Expressions, equations, and relationships. The student applies mathematical process standards to solve one-variable equations and inequalities. The student is expected to:
- (A) model and solve one-variable, two-step equations and inequalities;
 - (B) determine if the given value(s) make(s) one-variable, two-step equations and inequalities true; and
 - (C) write and solve equations using geometry concepts, including the sum of the angles in a triangle, and angle relationships.
- (14) Measurement and data. The student applies mathematical process standards to use statistical representations to analyze data. The student is expected to:
- (A) compare two groups of numeric data using comparative dot plots or box plots by comparing their shapes, centers, and spreads;
 - (B) use data from a random sample to make inferences about a population; and
 - (C) compare two populations based on data in random samples from these populations, including informal comparative inferences about differences between the two populations.

§111.24. Grade 8, Beginning with School Year 2013-2014.

(a) Introduction.

- (1) The desire to achieve educational excellence is the driving force behind the Texas essential knowledge and skills for mathematics, guided by the college and career readiness standards. By embedding statistics, probability, and finance, while focusing on fluency and solid understanding, Texas will lead the way in mathematics education and prepare all Texas students for the challenges they will face in the 21st century.
- (2) The process standards describe ways in which students are expected to engage in the content. The placement of the process standards at the beginning of the knowledge and skills listed for each grade and course is intentional. The process standards weave the other knowledge and skills together so that students may be successful problem solvers and use mathematics efficiently and effectively in daily life. The process standards are integrated at every grade level and course. When possible, students will apply mathematics to problems arising in everyday life, society, and the workplace. Students will use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution. Students will select appropriate tools such as real objects, manipulatives, paper and pencil, and technology and techniques such as mental math, estimation, and number sense to solve problems. Students will effectively communicate mathematical ideas, reasoning, and their implications using multiple representations such as symbols, diagrams, graphs, and language. Students will use mathematical relationships to generate solutions and make connections and predictions. Students will analyze mathematical relationships to connect and communicate mathematical ideas. Students will display, explain, or justify mathematical ideas and arguments using precise mathematical language in written or oral communication.
- (3) The primary focal areas in Grade 8 are proportionality; expressions, equations, relationships, and foundations of functions; and measurement and data. Students use concepts, algorithms, and properties of real numbers to explore mathematical relationships and to describe increasingly complex situations. Students use concepts of proportionality to explore, develop, and communicate mathematical relationships. Students use algebraic thinking to describe how a

change in one quantity in a relationship results in a change in the other. Students connect verbal, numeric, graphic, and symbolic representations of relationships, including equations and inequalities. Students begin to develop an understanding of functional relationships. Students use geometric properties and relationships, as well as spatial reasoning, to model and analyze situations and solve problems. Students communicate information about geometric figures or situations by quantifying attributes, generalize procedures from measurement experiences, and use the procedures to solve problems. Students use appropriate statistics, representations of data, and reasoning to draw conclusions, evaluate arguments, and make recommendations. While the use of all types of technology is important, the emphasis on algebra readiness skills necessitates the implementation of graphing technology.

(4) Statements that contain the word "including" reference content that must be mastered, while those containing the phrase "such as" are intended as possible illustrative examples.

(b) Knowledge and skills.

(1) Mathematical process standards. The student uses mathematical processes to acquire and demonstrate mathematical understanding. The student is expected to:

(A) apply mathematics to problems arising in everyday life, society, and the workplace;

(B) use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution;

(C) select tools, including real objects, manipulatives, paper and pencil, and technology as appropriate, and techniques, including mental math, estimation, and number sense as appropriate, to solve problems;

(D) communicate mathematical ideas, reasoning, and their implications using multiple representations, including symbols, diagrams, graphs, and language as appropriate;

(E) create and use representations to organize, record, and communicate mathematical ideas;

(F) analyze mathematical relationships to connect and communicate mathematical ideas; and

(G) display, explain, and justify mathematical ideas and arguments using precise mathematical language in written or oral communication.

(2) Number and operations. The student applies mathematical process standards to represent and use real numbers in a variety of forms. The student is expected to:

(A) extend previous knowledge of sets and subsets using a visual representation such as a Venn diagram to describe relationships between sets of real numbers;

(B) add, subtract, multiply, and divide rational numbers fluently;

(C) approximate the value of an irrational number and locate that rational number approximation on a number line, including π and square roots of numbers less than 225;

(D) convert between base-10 notation and scientific notation; and

(E) order a set of real numbers arising from mathematical and real-world contexts.

(3) Proportionality. The student applies mathematical process standards to use proportional relationships to describe dilations. The student is expected to:

(A) generalize that the ratio of corresponding sides of similar shapes are proportional, including a shape and its dilation;

(B) compare and contrast the attributes of a shape and its dilation(s) on a coordinate plane; and

- (C) use an algebraic representation to explain the effect of a given positive rational scale factor applied to two-dimensional figures on a coordinate plane with the origin as the center of dilation such as $(x, y) \rightarrow (0.5x, 0.5y)$.
- (4) Proportionality. The student applies mathematical process standards to explain proportional and non-proportional relationships involving slope. The student is expected to:
- (A) use similar right triangles to develop an understanding that slope, m , given as the rate comparing the change in y -values to the change in x -values, $(y_2 - y_1) / (x_2 - x_1)$, is the same for any two points (x_1, y_1) and (x_2, y_2) on the same line;
- (B) graph proportional relationships, interpreting the unit rate as the slope of the line that models the relationship; and
- (C) use data from a table or graph to determine the rate of change or slope and y -intercept in mathematical and real-world problems.
- (5) Proportionality. The student applies mathematical process standards to use proportional and non-proportional relationships to develop foundational concepts of functions. The student is expected to:
- (A) represent linear proportional situations with tables, graphs, and equations in the form of $y = kx$;
- (B) represent linear non-proportional situations with tables, graphs, and equations in the form of $y = mx + b$, where $b \neq 0$;
- (C) contrast bivariate sets of data that suggest a linear relationship with bivariate sets of data that do not suggest a linear relationship from a graphical representation;
- (D) use a trend line that approximates the linear relationship between bivariate sets of data to make predictions;
- (E) solve problems involving direct variation;
- (F) solve directly proportional problems;
- (G) distinguish between proportional and non-proportional situations using tables, graphs, and equations in the form $y = kx$ or $y = mx + b$, where $b \neq 0$;
- (H) identify functions using sets of ordered pairs, tables, mappings, and graphs;
- (I) identify examples of proportional and non-proportional functions that arise from mathematical and real-world problems; and
- (J) write an equation in the form $y = mx + b$ to model a linear relationship between two quantities using verbal, numerical, tabular, and graphical representations.
- (6) Expressions, equations, and relationships. The student applies mathematical process standards to develop mathematical relationships and make connections to geometric formulas. The student is expected to:
- (A) describe the volume formula $V = Bh$ of a cylinder in terms of its base area and its height;
- (B) model the relationship between the volume of a cylinder and a cone having both congruent bases and heights and connect that relationship to the formulas; and
- (C) use models and diagrams to explain the Pythagorean theorem.
- (7) Expressions, equations, and relationships. The student applies mathematical process standards to use geometry to solve problems. The student is expected to:
- (A) solve problems involving the volume of cylinders, cones, and spheres;

- (B) use previous knowledge of surface area to make connections to the formulas for lateral and total surface area and determine solutions for problems involving rectangular prisms, triangular prisms, and cylinders;
 - (C) use the Pythagorean Theorem and its converse to solve problems; and
 - (D) determine the distance between two points on a coordinate plane using the Pythagorean Theorem.
- (8) Expressions, equations, and relationships. The student applies mathematical process standards to use one-variable equations in problem situations. The student is expected to:
- (A) write one-variable equations with variables on both sides that represent problems using rational number coefficients and constants;
 - (B) write a corresponding real-world problem when given a one-variable equation with variables on both sides of the equal sign using rational number coefficients and constants;
 - (C) model and solve one-variable equations with variables on both sides of the equal sign that represent mathematical and real-world problems using rational number coefficients and constants;
 - (D) write and solve equations using geometry concepts, including the angle relationships when parallel lines are cut by a transversal; and
 - (E) write and solve equations using geometry concepts, including the properties of side lengths and angles in quadrilaterals.
- (9) Expressions, equations, and relationships. The student applies mathematical process standards to use multiple representations to develop foundational concepts of simultaneous linear equations. The student is expected to identify and verify the values of x and y that simultaneously satisfy two linear equations in the form $y = mx + b$ from the intersections of the graphed equations.
- (10) Two-dimensional shapes. The student applies mathematical process standards to develop transformational geometry concepts. The student is expected to:
- (A) generalize the properties of orientation and congruence of rotations, reflections, translations, and dilations of two-dimensional shapes on a coordinate plane;
 - (B) differentiate between transformations that preserve congruence and those that do not;
 - (C) explain the effect of translations, reflections over the x - or y -axis, and rotations limited to 90° , 180° , 270° , and 360° as applied to two-dimensional shapes on a coordinate plane using an algebraic representation such as $(x, y) \rightarrow (x + 2, y + 2)$; and
 - (D) model the effect on linear and area measurements of dilated two-dimensional shapes.
- (11) Measurement and data. The student applies mathematical process standards to use statistical procedures to describe data. The student is expected to:
- (A) construct a scatterplot and describe the observed trend such as positive trend, negative trend, and no trend to address questions of association such as linear, non-linear, and no association between bivariate data;
 - (B) determine the mean absolute deviation and use this quantity as a measure of the average distance data are from the mean using a data set of no more than 10 data points; and
 - (C) simulate generating random samples of the same size from a population with known characteristics to develop the notion of a random sample being representative of the population from which it was selected.

Subchapter C. High School

§111.38. Implementation of Texas Essential Knowledge and Skills for Mathematics, High School, Beginning with School Year 2013-2014.

The provisions of §§111.39-111.45 of this subchapter shall be implemented by school districts beginning with the 2013-2014 school year. This implementation date shall supersede any other implementation dates found in this subchapter.

§111.39. Algebra I, Beginning with School Year 2013-2014 (One Credit).

- (a) General requirements. Students shall be awarded one credit for successful completion of this course. This course is recommended for students in Grade 8 or 9. Prerequisite: Mathematics, Grade 8 or its equivalent.
- (b) Introduction.
- (1) The desire to achieve educational excellence is the driving force behind the Texas essential knowledge and skills for mathematics, guided by the college and career readiness standards. By embedding statistics, probability, and finance, while focusing on fluency and solid understanding, Texas will lead the way in mathematics education and prepare all Texas students for the challenges they will face in the 21st century.
 - (2) The process standards describe ways in which students are expected to engage in the content. The placement of the process standards at the beginning of the knowledge and skills listed for each grade and course is intentional. The process standards weave the other knowledge and skills together so that students may be successful problem solvers and use mathematics efficiently and effectively in daily life. The process standards are integrated at every grade level and course. When possible, students will apply mathematics to problems arising in everyday life, society, and the workplace. Students will use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution. Students will select appropriate tools such as real objects, manipulatives, paper and pencil, and technology and techniques such as mental math, estimation, and number sense to solve problems. Students will effectively communicate mathematical ideas, reasoning, and their implications using multiple representations such as symbols, diagrams, graphs, and language. Students will use mathematical relationships to generate solutions and make connections and predictions. Students will analyze mathematical relationships to connect and communicate mathematical ideas. Students will display, explain, or justify mathematical ideas and arguments using precise mathematical language in written or oral communication.
 - (3) In Algebra I, students will build on the knowledge and skills for mathematics in Grades 6-8, which provide a foundation in linear relationships, number and operations, and proportionality. Students will study linear, quadratic, and exponential functions and their related transformations, equations, and associated solutions. Students will connect functions and their associated solutions in both mathematical and real-world situations. Students will use technology to collect and explore data and analyze statistical relationships. In addition, students will study polynomials of degree one and two, radical expressions, sequences, and laws of exponents. Students will generate and solve linear systems with two equations and two variables and will create new functions through transformations
 - (4) Statements that contain the word "including" reference content that must be mastered, while those containing the phrase "such as" are intended as possible illustrative examples.
- (c) Knowledge and skills.
- (1) Mathematical process standards. The student uses mathematical processes to acquire and demonstrate mathematical understanding. The student is expected to:
 - (A) apply mathematics to problems arising in everyday life, society, and the workplace;

- (B) use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution;
 - (C) select tools, including real objects, manipulatives, paper and pencil, and technology as appropriate, and techniques, including mental math, estimation, and number sense as appropriate, to solve problems;
 - (D) communicate mathematical ideas, reasoning, and their implications using multiple representations, including symbols, diagrams, graphs, and language as appropriate;
 - (E) create and use representations to organize, record, and communicate mathematical ideas;
 - (F) analyze mathematical relationships to connect and communicate mathematical ideas; and
 - (G) display, explain, and justify mathematical ideas and arguments using precise mathematical language in written or oral communication.
- (2) Linear functions, equations, and inequalities. The student applies the mathematical process standards when using properties of linear functions to write and represent in multiple ways, with and without technology, linear equations, inequalities, and systems of equations. The student is expected to:
- (A) determine the domain and range of a linear function in mathematical problems and determine reasonable domain and range values for real-world situations, both continuous and discrete;
 - (B) write linear equations in two variables in various forms, including $y = mx + b$, $Ax + By = C$, and $y - y_1 = m(x - x_1)$, given one point and the slope and given two points;
 - (C) write linear equations in two variables given a table of values, a graph, and a verbal description;
 - (D) write and solve equations involving direct variation;
 - (E) write linear equations in two variables that contain a given point and are parallel to a given line;
 - (F) write linear equations in two variables that contain a given point and are perpendicular to a given line;
 - (G) write linear equations in two variables that are parallel and lines that are perpendicular to the x - and to the y -axis and determine whether their slopes are zero or undefined;
 - (H) write linear inequalities in two variables given a table of values, a graph, and a verbal description; and
 - (I) write systems of two linear equations given a table of values, a graph, and a verbal description.
- (3) Linear functions, equations, and inequalities. The student applies the mathematical process standards when using graphs of linear functions, key features, and related transformations to represent in multiple ways and solve, with and without technology, equations, inequalities, and systems of equations. The student is expected to:
- (A) determine the slope of a line given a table of values, a graph, two points on the line, and an equation written in various forms, including $y = mx + b$, $Ax + By = C$, and $y - y_1 = m(x - x_1)$;
 - (B) calculate the rate of change of a linear function represented tabularly, graphically, and algebraically over a specified interval within mathematical and real-world problems;
 - (C) graph linear functions on the coordinate plane and identify key features, including x -intercept, y -intercept, zeros, and slope, in mathematical and real-world problems;

- (D) graph the solution set of linear inequalities in two variables on the coordinate plane;
 - (E) determine the effects on the graph of the parent function $f(x) = x$ when $f(x)$ is replaced by $af(x)$, $f(x) + d$, $f(x - c)$, $f(bx)$ for specific values of a , b , c , and d ;
 - (F) graph systems of two linear equations in two variables on the coordinate plane and determine the solutions if they exist;
 - (G) estimate graphically the solutions to systems of two linear equations with two variables in real-world problems; and
 - (H) graph the solution set of systems of two linear inequalities in two variables on the coordinate plane.
- (4) Linear functions, equations, and inequalities. The student applies the mathematical process standards to formulate statistical relationships and evaluate their reasonableness based on real-world data. The student is expected to:
- (A) calculate, using technology, the correlation coefficient between two quantitative variables and interpret this quantity as a measure of the strength of the linear association;
 - (B) compare and contrast association and causation in real-world problems; and
 - (C) write, with and without technology, linear functions that provide a reasonable fit to data to estimate solutions and make predictions for real-world problems.
- (5) Linear functions, equations, and inequalities. The student applies the mathematical process standards to solve, with and without technology, linear equations and evaluate the reasonableness of their solutions. The student is expected to:
- (A) solve linear equations in one variable, including those for which the application of the distributive property is necessary and for which variables are included on both sides;
 - (B) solve linear inequalities in one variable, including those for which the application of the distributive property is necessary and for which variables are included on both sides; and
 - (C) solve, using substitution and Gaussian elimination, systems of two linear equations with two variables for mathematical and real-world problems.
- (6) Quadratic functions and equations. The student applies the mathematical process standards when using properties of quadratic functions to write and represent in multiple ways, with and without technology, quadratic equations. The student is expected to:
- (A) determine the domain and range of quadratic functions;
 - (B) write equations of quadratic functions given the vertex and another point on the graph, write the equation in vertex form ($f(x) = a(x - h)^2 + k$), and rewrite the equation from vertex form to standard form ($f(x) = ax^2 + bx + c$); and
 - (C) write quadratic functions when given real solutions and graphs of their related equations.
- (7) Quadratic functions and equations. The student applies the mathematical process standards when using graphs of quadratic functions and their related transformations to represent in multiple ways and determine, with and without technology, the solutions to equations. The student is expected to:
- (A) graph quadratic functions on the coordinate plane and use the graph to identify key attributes, if possible, including x -intercept, y -intercept, zeros, maximum value, minimum values, vertex, and the equation of the axis of symmetry;
 - (B) describe the relationship between the linear factors of quadratic expressions and the zeros of their associated quadratic functions; and
 - (C) determine the effects on the graph of the parent function $f(x) = x^2$ when $f(x)$ is replaced by $af(x)$, $f(x) + d$, $f(x - c)$, $f(bx)$ for specific values of a , b , c , and d .

- (8) Quadratic functions and equations. The student applies the mathematical process standards to solve, with and without technology, quadratic equations and evaluate the reasonableness of their solutions. The student formulates statistical relationships and evaluates their reasonableness based on real-world data. The student is expected to:
- (A) solve quadratic equations, having real solutions by factoring, taking square roots, completing the square, and applying the quadratic formula; and
 - (B) write, using technology, quadratic functions that provide a reasonable fit to data to estimate solutions and make predictions for real-world problems.
- (9) Exponential functions and equations. The student applies the mathematical process standards when using properties of exponential functions and their related transformations to write, graph, and represent in multiple ways exponential equations and evaluate, with and without technology, the reasonableness of their solutions. The student formulates statistical relationships and evaluates their reasonableness based on real-world data. The student is expected to:
- (A) determine the domain and range of exponential functions of the form $f(x) = ab^x$;
 - (B) interpret the meaning of the values of a and b in exponential functions of the form $f(x) = ab^x$ in real-world problems;
 - (C) write exponential functions in the form $f(x) = ab^x$ (where b is a rational number) to describe problems arising from mathematical and real-world situations, including growth and decay;
 - (D) graph exponential functions that model growth and decay and identify key features, including y -intercept and asymptotes, in mathematical and real-world problems; and
 - (E) write, using technology, exponential functions that provide a reasonable fit to data and make predictions for real-world problems.
- (10) Number and algebraic methods. The student applies the mathematical process standards and algebraic methods to rewrite in equivalent forms and perform operations on polynomial expressions. The student is expected to:
- (A) add and subtract polynomials of degree one and degree two;
 - (B) multiply polynomials of degree one and degree two;
 - (C) determine the quotient of a polynomial of degree one and polynomial of degree two when divided by a polynomial of degree one and polynomial of degree two;
 - (D) rewrite polynomial expressions of degree one and degree two in equivalent forms using the distributive property such as rewriting $(4x)(x - 2)$ as $(4x)(x) - (4x)(2)$, then writing the expression as $4x^2 - 8x$, or $4x^2 - 8x$ to $(4x)(x) - (4x)(2)$, and then factoring the result as $(4x)(x - 2)$;
 - (E) factor, if possible, trinomials with real factors in the form $ax^2 + bx + c$, including perfect square trinomials of degree two; and
 - (F) decide if a binomial can be written as the difference of two squares and, if possible, use the structure of a difference of two squares to rewrite the binomial such as rewriting the expression $49x^4 - y^4$ to $(7x^2)^2 - (y^2)^2$ and then factoring the binomial as $(7x^2 + y^2)(7x^2 - y^2)$.
- (11) Number and algebraic methods. The student applies the mathematical process standards and algebraic methods to rewrite algebraic expressions into equivalent forms. The student is expected to:
- (A) simplify numerical radical expressions involving square roots; and
 - (B) simplify numeric and algebraic expressions using the laws of exponents, including integral and rational exponents.

- (12) Number and algebraic methods. The student applies the mathematical process standards and algebraic methods to write, solve, analyze, and evaluate equations, relations, and functions. The student is expected to:
- (A) decide whether relations represented verbally, tabularly, graphically, and symbolically define a function;
 - (B) evaluate functions, expressed in function notation, given one or more elements in their domains;
 - (C) identify terms of arithmetic and geometric sequences when the sequences are given in function form and given in recursive form;
 - (D) write a formula for the n^{th} term of arithmetic and geometric sequences, given the value of several of their terms; and
 - (E) solve mathematic and scientific formulas, and other literal equations, for a specified variable.

§111.40. Algebra II, Beginning with School Year 2013-2014 (One-Half to One Credit).

- (a) General requirements. Students shall be awarded one-half to one credit for successful completion of this course. Prerequisite: Algebra I.
- (b) Introduction.
 - (1) The desire to achieve educational excellence is the driving force behind the Texas essential knowledge and skills for mathematics, guided by the college and career readiness standards. By embedding statistics, probability, and finance, while focusing on fluency and solid understanding, Texas will lead the way in mathematics education and prepare all Texas students for the challenges they will face in the 21st century.
 - (2) The process standards describe ways in which students are expected to engage in the content. The placement of the process standards at the beginning of the knowledge and skills listed for each grade and course is intentional. The process standards weave the other knowledge and skills together so that students may be successful problem solvers and use mathematics efficiently and effectively in daily life. The process standards are integrated at every grade level and course. When possible, students will apply mathematics to problems arising in everyday life, society, and the workplace. Students will use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution. Students will select appropriate tools such as real objects, manipulatives, paper and pencil, and technology and techniques such as mental math, estimation, and number sense to solve problems. Students will effectively communicate mathematical ideas, reasoning, and their implications using multiple representations such as symbols, diagrams, graphs, and language. Students will use mathematical relationships to generate solutions and make connections and predictions. Students will analyze mathematical relationships to connect and communicate mathematical ideas. Students will display, explain, or justify mathematical ideas and arguments using precise mathematical language in written or oral communication.
 - (3) In Algebra II, students will build on the knowledge and skills in Kindergarten-Grade 8 and Algebra I. Students will broaden their knowledge of quadratic functions, exponential functions, and systems of equations. Students will study logarithmic, square root, cubic, cube root, absolute value, rational functions, and their related equations. Students connect functions to their inverses and associated equations and solutions in both mathematical and real-world situations. In addition, students will extend their knowledge of data analysis and numeric and algebraic methods.
 - (4) Statements that contain the word "including" reference content that must be mastered, while those containing the phrase "such as" are intended as possible illustrative examples.
- (c) Knowledge and skills.

- (1) Mathematical process standards. The student uses mathematical processes to acquire and demonstrate mathematical understanding. The student is expected to:
- (A) apply mathematics to problems arising in everyday life, society, and the workplace;
 - (B) use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution;
 - (C) select tools, including real objects, manipulatives, paper and pencil, and technology as appropriate, and techniques, including mental math, estimation, and number sense as appropriate, to solve problems;
 - (D) communicate mathematical ideas, reasoning, and their implications using multiple representations, including symbols, diagrams, graphs, and language as appropriate;
 - (E) create and use representations to organize, record, and communicate mathematical ideas;
 - (F) analyze mathematical relationships to connect and communicate mathematical ideas; and
 - (G) display, explain, or justify mathematical ideas and arguments using precise mathematical language in written or oral communication.
- (2) Attributes of functions and their inverses. The student applies mathematical processes to understand that functions have distinct key attributes and understand the relationship between a function and its inverse. The student is expected to:
- (A) graph the functions $f(x) = \sqrt{x}$, $f(x) = \frac{1}{x}$, $f(x) = x^3$, $f(x) = \sqrt[3]{x}$, $f(x) = b^x$, and $f(x) = \log_b(x)$ where b is 2, 10, and e , and, when applicable, analyze the key attributes such as domain, range, intercepts, symmetries, asymptotic behavior, and relative maxima and minima given an interval;
 - (B) graph and write the inverse of a function;
 - (C) describe and analyze the relationship between a function and its inverse (quadratic and square root, logarithmic and exponential), including the restrictions on domains and ranges; and
 - (D) use the composition of two functions, including the necessary restrictions on the domain, to determine if the functions are inverses of each other.
- (3) Systems of equations and inequalities. The student applies mathematical processes to formulate systems of equations and inequalities, use a variety of methods to solve, and analyze reasonableness of solutions. The student is expected to:
- (A) formulate systems of equations, including systems consisting of three linear equations in three variables and systems consisting of two equations, the first linear and the second quadratic;
 - (B) solve systems of three linear equations in three variables by methods such as elimination, using technology with matrices, and substitution;
 - (C) solve, algebraically, systems of two equations in two variables consisting of a linear equation and a quadratic equation;
 - (D) determine the reasonableness of solutions to systems of a linear equation and a quadratic equation in two variables;
 - (E) formulate systems of at least two linear inequalities in two variables;
 - (F) solve systems of two or more linear inequalities in two variables; and
 - (G) determine possible solutions in the solution set of systems of two or more linear inequalities in two variables.

- (4) Quadratic and square root functions, equations, and inequalities. The student applies mathematical processes to understand that quadratic and square root functions and quadratic inequalities can be used to model situations, solve problems, and make predictions. The student is expected to:
- (A) write the quadratic function given three specified points in the plane;
 - (B) write the equation of a parabola using given attributes, including vertex, focus, directrix, axis of symmetry, and direction of opening;
 - (C) determine the effect on the graph of $f(x) = \sqrt{x}$ when $f(x)$ is replaced by $af(x)$, $f(x) + d$, $f(bx)$, and $f(x - c)$ for specific positive and negative values of a , b , c , and d ;
 - (D) transform a quadratic function $f(x) = ax^2 + bx + c$ to the form $f(x) = a(x - h)^2 + k$ to identify the different attributes of $f(x)$;
 - (E) formulate quadratic and square root equations;
 - (F) solve quadratic and square root equations;
 - (G) identify extraneous solutions of square root equations; and
 - (H) solve quadratic inequalities.
- (5) Exponential and logarithmic functions and equations. The student applies mathematical processes to understand that exponential and logarithmic functions can be used to model situations and solve problems. The student is expected to:
- (A) determine the effects on the key attributes on the graphs of $f(x) = b^x$ and $f(x) = \log_b(x)$ where b is 2, 10, and e when $f(x)$ is replaced by $af(x)$, $f(x) + d$, and $f(x - c)$ for specific positive and negative values of a , c , and d ;
 - (B) formulate exponential and logarithmic equations that model real-world situations;
 - (C) rewrite exponential equations as their corresponding logarithmic equations and logarithmic equations as their corresponding exponential equations;
 - (D) solve exponential equations of the form $y = ab^x$ where a is a nonzero real number and b is greater than zero and not equal to one and single logarithmic equations have real solutions; and
 - (E) determine the reasonableness of a solution to a logarithmic equation.
- (6) Cubic, cube root, absolute value and rational functions, equations, and inequalities. The student applies mathematical processes to understand that cubic, cube root, absolute value and rational functions, equations, and inequalities can be used to model situations, solve problems, and make predictions. The student is expected to:
- (A) analyze the effect on the graphs of $f(x) = x^3$ and $f(x) = \sqrt[3]{x}$ when $f(x)$ is replaced by $af(x)$, $f(bx)$, $f(x - c)$, and $f(x) + d$ for specific positive and negative values of a , b , c , and d ;
 - (B) solve cube root equations;
 - (C) analyze the effect on the graphs of $f(x) = |x|$ when $f(x)$ is replaced by $af(x)$, $f(bx)$, $f(x - c)$, and $f(x) + d$ for specific positive and negative values of a , b , c , and d ;
 - (D) formulate absolute value linear equations that model real-world situations;
 - (E) solve absolute value linear equations;
 - (F) solve absolute value linear inequalities;
 - (G) analyze the effect on the graphs of $f(x) = \frac{1}{x}$ when $f(x)$ is replaced by $af(x)$, $f(bx)$, $f(x - c)$, and $f(x) + d$ for specific positive and negative values of a , b , c , and d ;
 - (H) formulate rational equations that model real-world situations;
 - (I) solve rational equations that have real solutions;

- (J) determine the reasonableness of a solution to a rational equation;
 - (K) determine the restrictions on the domain of a rational function; and
 - (L) formulate and solve equations involving inverse variation.
- (7) Number and algebraic methods. The student applies mathematical processes to simplify and perform operations on expressions and to solve equations. The student is expected to:
- (A) add, subtract, and multiply complex numbers;
 - (B) add, subtract, and multiply polynomials;
 - (C) determine the quotient of a polynomial of degree three and of degree four when divided by a polynomial of degree one and of degree two;
 - (D) determine the linear factors of a polynomial function of degree three and of degree four using algebraic methods such as the Remainder Theorem;
 - (E) determine linear and quadratic factors of a polynomial expression of degree three and of degree four, including factoring the sum and difference of two cubes and factoring by grouping;
 - (F) determine the sum, difference, product, and quotient of rational expressions with integral exponents of degree one and of degree two;
 - (G) rewrite radical expressions that contain variables to equivalent forms;
 - (H) solve equations involving rational exponents; and
 - (I) write the domain and range of a function in interval notation.
- (8) Data. The student applies mathematical processes to analyze data, select appropriate models, write corresponding functions, and make predictions. The student is expected to:
- (A) analyze data to select the appropriate model from among linear, quadratic, and exponential models;
 - (B) use regression methods available through technology to write a linear function, a quadratic function, and an exponential function from a given set of data; and
 - (C) predict and make decisions and critical judgments from a given set of data using linear, quadratic, and exponential models.

§111.41. Geometry, Beginning with School Year 2013-2014 (One Credit).

- (a) General requirements. Students shall be awarded one credit for successful completion of this course. Prerequisite: Algebra I.
- (b) Introduction.
 - (1) The desire to achieve educational excellence is the driving force behind the Texas essential knowledge and skills for mathematics, guided by the college and career readiness standards. By embedding statistics, probability, and finance, while focusing on fluency and solid understanding, Texas will lead the way in mathematics education and prepare all Texas students for the challenges they will face in the 21st century.
 - (2) The process standards describe ways in which students are expected to engage in the content. The placement of the process standards at the beginning of the knowledge and skills listed for each grade and course is intentional. The process standards weave the other knowledge and skills together so that students may be successful problem solvers and use mathematics efficiently and effectively in daily life. The process standards are integrated at every grade level and course. When possible, students will apply mathematics to problems arising in everyday life, society, and the workplace. Students will use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and

evaluating the problem-solving process and the reasonableness of the solution. Students will select appropriate tools such as real objects, manipulatives, paper and pencil, and technology and techniques such as mental math, estimation, and number sense to solve problems. Students will effectively communicate mathematical ideas, reasoning, and their implications using multiple representations such as symbols, diagrams, graphs, and language. Students will use mathematical relationships to generate solutions and make connections and predictions. Students will analyze mathematical relationships to connect and communicate mathematical ideas. Students will display, explain, or justify mathematical ideas and arguments using precise mathematical language in written or oral communication.

- (3) In Geometry, students will build on the knowledge and skills for mathematics in Kindergarten-Grade 8 and Algebra I to strengthen their mathematical reasoning skills in geometric contexts. Within the course, students will begin to focus on more precise terminology, symbolic representations, and the development of proofs. Students will explore concepts covering coordinate and transformational geometry; logical argument and constructions; proof and congruence; similarity, proof, and trigonometry; two- and three-dimensional figures; circles; and probability. Students will connect previous knowledge from Algebra I to Geometry through the coordinate and transformational geometry strand. In the logical arguments and constructions strand, students are expected to create formal constructions using a straight edge and compass. Though this course is primarily Euclidean geometry, students should complete the course with an understanding that non-Euclidean geometries exist. In proof and congruence, students will use deductive reasoning to justify, prove and apply theorems about geometric figures. Throughout the standards, the term "prove" means a formal proof to be shown in a paragraph, a flow chart, or two-column formats. Proportionality is the unifying component of the similarity, proof, and trigonometry strand. Students will use their proportional reasoning skills to prove and apply theorems and solve problems in this strand. The two- and three-dimensional figure strand focuses on the application of formulas in multi-step situations since students have developed background knowledge in two- and three-dimensional figures. Using patterns to identify geometric properties, students will apply theorems about circles to determine relationships between special segments and angles in circles. Due to the emphasis of probability and statistics in the college and career readiness standards, standards dealing with probability have been added to the geometry curriculum to ensure students have proper exposure to these topics before pursuing their post-secondary education.
- (4) These standards are meant to provide clarity and specificity in regards to the content covered in the high school geometry course. These standards are not meant to limit the methodologies used to convey this knowledge to students. Though the standards are written in a particular order, they are not necessarily meant to be taught in the given order. In the standards, the phrase "to solve problems" includes both contextual and non-contextual problems unless specifically stated.
- (5) Statements that contain the word "including" reference content that must be mastered, while those containing the phrase "such as" are intended as possible illustrative examples.

(c) Knowledge and skills.

- (1) Mathematical process standards. The student uses mathematical processes to acquire and demonstrate mathematical understanding. The student is expected to:
- (A) apply mathematics to problems arising in everyday life, society, and the workplace;
- (B) use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution;
- (C) select tools, including real objects, manipulatives, paper and pencil, and technology as appropriate, and techniques, including mental math, estimation, and number sense as appropriate, to solve problems;
- (D) communicate mathematical ideas, reasoning, and their implications using multiple representations, including symbols, diagrams, graphs, and language as appropriate;

- (E) create and use representations to organize, record, and communicate mathematical ideas;
 - (F) analyze mathematical relationships to connect and communicate mathematical ideas; and
 - (G) display, explain, and justify mathematical ideas and arguments using precise mathematical language in written or oral communication.
- (2) Coordinate and transformational geometry. The student uses the process skills to understand the connections between algebra and geometry and uses the one- and two-dimensional coordinate systems to verify geometric conjectures. The student is expected to:
- (A) determine the coordinates of a point that is a given fractional distance less than one from one end of a line segment to the other in one- and two-dimensional coordinate systems, including finding the midpoint;
 - (B) derive and use the distance, slope, and midpoint formulas to verify geometric relationships, including congruence of segments and parallelism or perpendicularity of pairs of lines; and
 - (C) determine an equation of a line parallel or perpendicular to a given line that passes through a given point.
- (3) Coordinate and transformational geometry. The student uses the process skills to generate and describe rigid transformations (translation, reflection, and rotation) and non-rigid transformations (dilations that preserve similarity and reductions and enlargements that do not preserve similarity). The student is expected to:
- (A) describe and perform transformations of figures in a plane using coordinate notation, such as $(x, y) \rightarrow (-x, y)$;
 - (B) determine the image or pre-image of a given two-dimensional figure under a composition of rigid transformations, a composition of non-rigid transformations, and a composition of both, including dilations where the center can be any point in the plane;
 - (C) identify the sequence of transformations that will carry a given pre-image onto an image on and off the coordinate plane; and
 - (D) identify and distinguish between reflectional and rotational symmetry in a plane figure.
- (4) Logical argument and constructions. The student uses the process skills with inductive reasoning to understand geometric relationships. The student is expected to:
- (A) distinguish between undefined terms, definitions, postulates, conjectures, and theorems;
 - (B) identify and determine the validity of the converse, inverse, and contrapositive of a conditional statement and recognize the connection between a biconditional statement and a true conditional statement with a true converse;
 - (C) verify that a conjecture is false using a counterexample; and
 - (D) compare geometric relationships between Euclidean and spherical geometries, including parallel lines and the sum of the angles in a triangle.
- (5) Logical argument and constructions. The student uses constructions to validate conjectures about geometric figures. The student is expected to:
- (A) investigate patterns to make conjectures about geometric relationships, including angles formed by parallel lines cut by a transversal, criteria required for triangle congruence, special segments of triangles, diagonals of quadrilaterals, interior and exterior angles of polygons, and special segments and angles of circles choosing from a variety of tools such as compass and straightedge, paper folding, and dynamic geometric software;
 - (B) construct congruent segments, congruent angles, a segment bisector, an angle bisector, perpendicular lines, the perpendicular bisector of a line segment, and a line parallel to a given line through a point not on a line using a compass and a straightedge;

- (C) use the constructions of congruent segments, congruent angles, angle bisectors, and perpendicular bisectors to make conjectures about geometric relationships; and
 - (D) verify the Triangle Inequality theorem using constructions and apply the theorem to solve problems.
- (6) Proof and congruence. The student uses the process skills with deductive reasoning to prove and apply theorems by using a variety of methods such as coordinate, transformational, and axiomatic and formats such as two-column, paragraph, and flow chart. The student is expected to:
- (A) prove theorems about angles formed by the intersection of lines and line segments, including vertical angles, angles formed by parallel lines cut by a transversal, and equidistance between the endpoints of a segment and points on its perpendicular bisector, and apply these relationships to solve problems;
 - (B) prove two triangles are congruent by applying the Side-Angle-Side, Angle-Side-Angle, Side-Side-Side, Angle-Angle-Side, and Hypotenuse-Leg congruence conditions;
 - (C) apply the definition of congruence, in terms of rigid transformations, to identify congruent figures and their corresponding sides and angles;
 - (D) prove theorems about the relationships in triangles, including the sum of interior angles, base angles of isosceles triangles, midsegments, and medians, and apply these relationships to solve problems; and
 - (E) prove a quadrilateral is a parallelogram, rectangle, square, or rhombus using opposite sides, opposite angles, or diagonals and apply these relationships to solve problems.
- (7) Similarity, proof, and trigonometry. The student uses the process skills in applying similarity to solve problems. The student is expected to:
- (A) apply the definition of similarity in terms of a dilation to identify similar figures and their proportional sides and the congruent corresponding angles; and
 - (B) apply the Angle-Angle criterion to verify similar triangles and apply the proportionality of the corresponding sides to solve problems.
- (8) Similarity, proof, and trigonometry. The student uses the process skills with deductive reasoning to prove and apply theorems by using a variety of methods such as coordinate, transformational, and axiomatic and formats such as two-column, paragraph, and flow chart. The student is expected to:
- (A) prove theorems about similar triangles, including the Triangle Proportionality theorem, and apply these theorems to solve problems; and
 - (B) identify and apply the relationships that exist when an altitude is drawn to the hypotenuse of a right triangle, including the geometric mean, to solve problems.
- (9) Similarity, proof, and trigonometry. The student uses the process skills to understand and apply relationships in right triangles. The student is expected to:
- (A) determine the lengths of sides and measures of angles in a right triangle by applying the trigonometric ratios sine, cosine, and tangent to solve problems; and
 - (B) apply the relationships in special right triangles 30° - 60° - 90° and 45° - 45° - 90° and the Pythagorean theorem, including Pythagorean triples, to solve problems.
- (10) Two-dimensional and three-dimensional figures. The student uses the process skills to recognize characteristics and dimensional changes of two- and three-dimensional figures. The student is expected to:
- (A) identify the shapes of two-dimensional cross-sections of prisms, pyramids, cylinders, cones, and spheres and identify three-dimensional objects generated by rotations of two-dimensional shapes; and

- (B) determine and describe how changes in the linear dimensions of a shape affect its perimeter, area, surface area, or volume, including proportional and non-proportional dimensional change.
- (11) Two-dimensional and three-dimensional figures. The student uses the process skills in the application of formulas to determine measures of two- and three-dimensional figures. The student is expected to:
- (A) apply the formula for the area of regular polygons to solve problems using appropriate units of measure;
- (B) determine the area of composite two-dimensional figures comprised of a combination of triangles, parallelograms, trapezoids, kites, regular polygons, or sectors of circles to solve problems using appropriate units of measure;
- (C) apply the formulas for the total and lateral surface area of three-dimensional figures, including prisms, pyramids, cones, cylinders, spheres, and composite figures, to solve problems using appropriate units of measure; and
- (D) apply the formulas for the volume of three-dimensional figures, including prisms, pyramids, cones, cylinders, spheres, and composite figures, to solve problems using appropriate units of measure.
- (12) Circles. The student uses the process skills to understand geometric relationships and apply theorems and equations about circles. The student is expected to:
- (A) apply theorems about circles, including relationships among angles, radii, chords, tangents, and secants, to solve non-contextual problems;
- (B) apply the proportional relationship between the measure of an arc length of a circle and the circumference of the circle to solve problems;
- (C) apply the proportional relationship between the measure of the area of a sector of a circle and the area of the circle to solve problems;
- (D) describe radian measure of an angle as the ratio of the length of an arc intercepted by a central angle and the radius of the circle; and
- (E) show that the equation of a circle with center at the origin and radius r is $x^2 + y^2 = r^2$ and determine the equation for the graph of a circle with radius r and center (h, k) , $(x - h)^2 + (y - k)^2 = r^2$.
- (13) Probability. The student uses the process skills to understand probability in real-world situations and how to apply independence and dependence of events. The student is expected to:
- (A) develop strategies to use permutations and combinations to solve contextual problems;
- (B) determine probabilities based on area to solve contextual problems;
- (C) identify whether two events are independent and compute the probability of the two events occurring together with or without replacement;
- (D) apply conditional probability in contextual problems; and
- (E) apply independence in contextual problems.

§111.42. Precalculus, Beginning with School Year 2013-2014 (One-Half to One Credit).

- (a) General requirements. Students shall be awarded one-half to one credit for successful completion of this course. Prerequisites: Algebra I, Geometry, and Algebra II.
- (b) Introduction.
- (1) The desire to achieve educational excellence is the driving force behind the Texas essential knowledge and skills for mathematics, guided by the college and career readiness standards. By

embedding statistics, probability, and finance, while focusing on fluency and solid understanding. Texas will lead the way in mathematics education and prepare all Texas students for the challenges they will face in the 21st century.

- (2) The process standards describe ways in which students are expected to engage in the content. The placement of the process standards at the beginning of the knowledge and skills listed for each grade and course is intentional. The process standards weave the other knowledge and skills together so that students may be successful problem solvers and use mathematics efficiently and effectively in daily life. The process standards are integrated at every grade level and course. When possible, students will apply mathematics to problems arising in everyday life, society, and the workplace. Students will use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution. Students will select appropriate tools such as real objects, manipulatives, paper and pencil, and technology and techniques such as mental math, estimation, and number sense to solve problems. Students will effectively communicate mathematical ideas, reasoning, and their implications using multiple representations such as symbols, diagrams, graphs, and language. Students will use mathematical relationships to generate solutions and make connections and predictions. Students will analyze mathematical relationships to connect and communicate mathematical ideas. Students will display, explain, or justify mathematical ideas and arguments using precise mathematical language in written or oral communication.
- (3) Precalculus is the preparation for calculus. The course approaches topics from a function point of view, where appropriate, and is designed to strengthen and enhance conceptual understanding and mathematical reasoning used when modeling and solving mathematical and real-world problems. Students systematically work with functions and their multiple representations. The study of Precalculus deepens students' mathematical understanding and fluency with algebra and trigonometry and extends their ability to make connections and apply concepts and procedures at higher levels. Students investigate and explore mathematical ideas, develop multiple strategies for analyzing complex situations, and use technology to build understanding, make connections between representations, and provide support in solving problems.
- (4) Statements that contain the word "including" reference content that must be mastered, while those containing the phrase "such as" are intended as possible illustrative examples.

(c) Knowledge and skills.

- (1) Mathematical process standards. The student uses mathematical processes to acquire and demonstrate mathematical understanding. The student is expected to:
- (A) apply mathematics to problems arising in everyday life, society, and the workplace;
 - (B) use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution;
 - (C) select tools, including real objects, manipulatives, paper and pencil, and technology as appropriate, and techniques, including mental math, estimation, and number sense as appropriate, to solve problems;
 - (D) communicate mathematical ideas, reasoning, and their implications using multiple representations, including symbols, diagrams, graphs, and language as appropriate;
 - (E) create and use representations to organize, record, and communicate mathematical ideas;
 - (F) analyze mathematical relationships to connect and communicate mathematical ideas; and
 - (G) display, explain, and justify mathematical ideas and arguments using precise mathematical language in written or oral communication.
- (2) Functions. The student uses process standards in mathematics to explore, describe, and analyze the attributes of functions. The student makes connections between multiple representations of

functions and algebraically constructs new functions. The student analyzes and uses functions to model real-world problems. The student is expected to:

- (A) use the composition of two functions to model and solve real-world problems;
 - (B) demonstrate that function composition is not always commutative;
 - (C) represent a given function as a composite function of two or more functions;
 - (D) describe symmetry of graphs of even and odd functions;
 - (E) determine an inverse function, when it exists, for a given function over its domain or a subset of its domain and represent the inverse using multiple representations;
 - (F) graph exponential logarithmic rational, polynomial, power, trigonometric, inverse trigonometric, and piecewise defined functions, including step functions;
 - (G) graph functions, including exponential, logarithmic, sine, cosine, rational, polynomial, and power functions and their transformations, including $af(x)$, $f(x) + d$, $f(x - c)$, $f(bx)$ for specific values of a , b , c , and d , in mathematical and real-world problems;
 - (H) graph $\arcsin x$ and $\arccos x$ and describe the limitations on the domain;
 - (I) determine and analyze the key features of exponential, logarithmic, rational, polynomial, power, trigonometric, inverse trigonometric, and piecewise defined functions, including step functions such as domain, range, symmetry, relative maximum, relative minimum, zeros, asymptotes, and intervals over which the function is increasing or decreasing;
 - (J) analyze and describe end behavior of functions, including exponential, logarithmic, rational, polynomial, and power functions, using infinity notation to communicate this characteristic in mathematical and real-world problems;
 - (K) analyze characteristics of rational functions and the behavior of the function around the asymptotes, including horizontal, vertical, and oblique asymptotes;
 - (L) determine various types of discontinuities in the interval $(-\infty, \infty)$ as they relate to functions such as rational and piecewise defined functions and explore the limitations of the graphing calculator as it relates to the behavior of the function around discontinuities;
 - (M) describe the left-sided behavior and the right-sided behavior of the graph of a function around discontinuities;
 - (N) analyze situations modeled by functions, including exponential, logarithmic, rational, polynomial, and power functions, to solve real-world problems such as problems involving growth and decay and optimization;
 - (O) develop and use a sinusoidal function that models a situation in mathematical and real-world problems; and
 - (P) determine the values of the trigonometric functions at the special angles and relate them in mathematical and real-world problems.
- (3) Relations and geometric reasoning. The student uses the process standards in mathematics to model and make connections between algebraic and geometric relations. The student is expected to:
- (A) graph a set of parametric equations;
 - (B) convert parametric equations into rectangular relations and convert rectangular relations into parametric equations;
 - (C) use parametric equations to model and solve mathematical and real-world problems;
 - (D) graph points in the polar coordinate system and convert between rectangular coordinates and polar coordinates;

- (E) graph polar equations such as cardioids, limaçons, or lemniscates by plotting points and using technology;
 - (F) determine the conic section formed when a plane intersects a double-napped cone;
 - (G) make connections between the locus definition of conic sections and their equations in rectangular coordinates;
 - (H) use the characteristics of an ellipse to write the equation of an ellipse with center (h, k) ; and
 - (I) use the characteristics of a hyperbola to write the equation of a hyperbola with center (h, k) .
- (4) Number and measure. The student uses process standards in mathematics to apply appropriate techniques, tools, and formulas to calculate measures in mathematical and real-world problems. The student is expected to:
- (A) determine the relationship between the unit circle and the definition of a periodic function to evaluate trigonometric functions in mathematical and real-world problems;
 - (B) describe the relationship between degree and radian measure on the unit circle;
 - (C) represent angles in radians or degrees based on the concept of rotation and find the measure of reference angles and angles in standard position;
 - (D) represent angles in radians or degrees based on the concept of rotation in mathematical and real-world problems, including linear and angular velocity;
 - (E) determine the value of trigonometric ratios of angles and solve problems involving trigonometric ratios in mathematical and real-world problems;
 - (F) use trigonometry in mathematical and real-world problems, including directional bearing;
 - (G) use the Law of Sines in mathematical and real-world problems;
 - (H) use the Law of Cosines in mathematical and real-world problems;
 - (I) use vectors to model situations involving magnitude and direction;
 - (J) represent the addition of vectors and the multiplication of a vector by a scalar geometrically and symbolically; and
 - (K) apply vector addition and multiplication of a vector by a scalar in mathematical and real-world problems.
- (5) Algebraic reasoning. The student uses process standards in mathematics to evaluate expressions, describe patterns, formulate models, and solve equations and inequalities using properties, procedures, or algorithms. The student is expected to:
- (A) represent finite sums and infinite series using sigma notation;
 - (B) evaluate finite sums and geometric series, when possible, written in sigma notation;
 - (C) represent arithmetic sequences and geometric sequences using recursive formulas;
 - (D) calculate the n^{th} term and the n^{th} partial sum of an arithmetic series in mathematical and real-world problems;
 - (E) represent arithmetic series and geometric series using sigma notation;
 - (F) calculate the n^{th} term of a geometric series, the n^{th} partial sum of a geometric series, and sum of an infinite geometric series when it exists;
 - (G) apply the Binomial Theorem for the expansion of $(a + b)^n$ in powers of a and b for a positive integer n , where a and b are any numbers;
 - (H) use the properties of logarithms to evaluate or transform logarithmic expressions;

- (I) generate and solve logarithmic equations in mathematical and real-world problems;
- (J) generate and solve exponential equations in mathematical and real-world problems;
- (K) solve polynomial equations with real coefficients by applying a variety of techniques such as factoring, graphical methods, or technology in mathematical and real-world problems;
- (L) solve polynomial inequalities with real coefficients by applying a variety of techniques such as factoring, graphical methods, or technology and write the solution set of the polynomial inequality in interval notation in mathematical and real-world problems;
- (M) solve rational inequalities with real coefficients by applying a variety of techniques such as factoring, graphical methods, or technology and write the solution set of the rational inequality in interval notation in mathematical and real-world problems;
- (N) use trigonometric identities such as reciprocal, quotient, Pythagorean, cofunctions, even/odd, and sum and difference identities for cosine and sine to simplify trigonometric expressions; and
- (O) generate and solve trigonometric equations in mathematical and real-world problems.

§111.43. Mathematical Models with Applications, Beginning with School Year 2013-2014 (One-Half to One Credit).

- (a) General requirements. Students can be awarded one-half to one credit for successful completion of this course. Prerequisite: Algebra I. This course must be taken before receiving credit for Algebra II.
- (b) Introduction.
 - (1) The desire to achieve educational excellence is the driving force behind the Texas essential knowledge and skills for mathematics, guided by the college and career readiness standards. By embedding statistics, probability, and finance, while focusing on fluency and solid understanding, Texas will lead the way in mathematics education and prepare all Texas students for the challenges they will face in the 21st century.
 - (2) The process standards describe ways in which students are expected to engage in the content. The placement of the process standards at the beginning of the knowledge and skills listed for each grade and course is intentional. The process standards weave the other knowledge and skills together so that students may be successful problem solvers and use mathematics efficiently and effectively in daily life. The process standards are integrated at every grade level and course. When possible, students will apply mathematics to problems arising in everyday life, society, and the workplace. Students will use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution. Students will select appropriate tools such as real objects, manipulatives, paper and pencil, and technology and techniques such as mental math, estimation, and number sense to solve problems. Students will effectively communicate mathematical ideas, reasoning, and their implications using multiple representations such as symbols, diagrams, graphs, and language. Students will use mathematical relationships to generate solutions and make connections and predictions. Students will analyze mathematical relationships to connect and communicate mathematical ideas. Students will display, explain, or justify mathematical ideas and arguments using precise mathematical language in written or oral communication.
 - (3) Mathematical Models with Applications is designed to build on the knowledge and skills for mathematics in Kindergarten-Grade 8 and Algebra I. This mathematics course provides a path for students to succeed in Algebra II and prepares them for various post-secondary choices. Students learn to apply mathematics through experiences in personal finance, science, engineering, fine arts, and social sciences. Students use algebraic, graphical, and geometric reasoning to recognize patterns and structure, model information, solve problems, and communicate solutions. Students will select from tools such as physical objects; manipulatives; technology, including graphing

calculators, data collection devices, and computers; and paper and pencil and from methods such as algebraic techniques, geometric reasoning, patterns, and mental math to solve problems.

(4) In Mathematical Models with Applications, students will use a mathematical modeling cycle to analyze problems, understand problems better, and improve decisions. A basic mathematical modeling cycle is summarized in this paragraph. The student will:

(A) represent:

(i) identify the variables in the problem and select those that represent essential features;

(ii) formulate a model by creating and selecting from representations such as geometric, graphical, tabular, algebraic, or statistical that describe the relationships between the variables;

(B) compute: analyze and perform operations on the relationships between the variables to draw conclusions;

(C) interpret: interpret the results of the mathematics in terms of the original problem;

(D) revise: confirm the conclusions by comparing the conclusions with the problem and revising as necessary; and

(E) report: report on the conclusions and the reasoning behind the conclusions.

(5) Statements that contain the word "including" reference content that must be mastered, while those containing the phrase "such as" are intended as possible illustrative examples.

(c) Knowledge and skills.

(1) Mathematical process standards. The student uses mathematical processes to acquire and demonstrate mathematical understanding. The student is expected to:

(A) apply mathematics to problems arising in everyday life, society, and the workplace;

(B) use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution;

(C) select tools, including real objects, manipulatives, paper and pencil, and technology as appropriate, and techniques, including mental math, estimation, and number sense as appropriate, to solve problems;

(D) communicate mathematical ideas, reasoning, and their implications using multiple representations, including symbols, diagrams, graphs, and language as appropriate;

(E) create and use representations to organize, record, and communicate mathematical ideas;

(F) analyze mathematical relationships to connect and communicate mathematical ideas; and

(G) display, explain, and justify mathematical ideas and arguments using precise mathematical language in written or oral communication.

(2) Mathematical modeling in personal finance. The student uses mathematical processes with graphical and numerical techniques to study patterns and analyze data related to personal finance. The student is expected to:

(A) use rates and linear functions to solve problems involving personal finance and budgeting, including compensations and deductions;

(B) solve problems involving personal taxes; and

(C) analyze data to make decisions about banking, including options for online banking, checking accounts, overdraft protection, processing fees, and debit card/ATM fees.

- (3) Mathematical modeling in personal finance. The student uses mathematical processes with algebraic formulas, graphs, and amortization modeling to solve problems involving credit. The student is expected to:
- (A) use formulas to generate tables to display series of payments for loan amortizations resulting from financed purchases;
 - (B) analyze personal credit options in retail purchasing and compare relative advantages and disadvantages of each option;
 - (C) use technology to create amortization models to investigate home financing and compare buying a home to renting a home; and
 - (D) use technology to create amortization models to investigate automobile financing and compare buying a vehicle to leasing a vehicle.
- (4) Mathematical modeling in personal finance. The student uses mathematical processes with algebraic formulas, numerical techniques, and graphs to solve problems related to financial planning. The student is expected to:
- (A) analyze and compare coverage options and rates in insurance;
 - (B) investigate and compare investment options, including stocks, bonds, annuities, certificates of deposit, and retirement plans; and
 - (C) analyze types of savings options involving simple and compound interest and compare relative advantages of these options.
- (5) Mathematical modeling in science and engineering. The student applies mathematical processes with algebraic techniques to study patterns and analyze data as it applies to science. The student is expected to:
- (A) use proportionality and inverse variation to describe physical laws such as Hook's Law, Newton's Second Law of Motion, and Boyle's Law;
 - (B) use exponential models available through technology to model growth and decay in areas such as population, biology, ecology, and chemistry, including radioactive decay; and
 - (C) use quadratic functions to model motion such as an object dropped, bounced, thrown, or kicked.
- (6) Mathematical modeling in science and engineering. The student applies mathematical processes with algebra and geometry to study patterns and analyze data as it applies to architecture and engineering. The student is expected to:
- (A) use similarity, geometric transformations, symmetry, and perspective drawings to describe mathematical patterns and structure in architecture;
 - (B) use scale factors with two-dimensional and three-dimensional objects to demonstrate proportional and non-proportional changes in surface area and volume as applied to fields such as engineering drawing, architecture, and construction;
 - (C) use the Pythagorean Theorem and special right-triangle relationships to calculate distances; and
 - (D) use trigonometric ratios to calculate distances and angle measures as applied to fields such as surveying, navigation, and orienteering.
- (7) Mathematical modeling in fine arts. The student uses mathematical processes with algebra and geometry to study patterns and analyze data as it applies to fine arts. The student is expected to:
- (A) use trigonometric ratios and functions available through technology to model periodic behavior in art and music;

- (B) use similarity, geometric transformations, symmetry, and perspective drawings to describe mathematical patterns and structure in art and photography;
 - (C) use geometric transformations, proportions, and periodic motion to describe mathematical patterns and structure in music; and
 - (D) use scale factors with two-dimensional and three-dimensional objects to demonstrate proportional and non-proportional changes in surface area and volume as applied to fields such as painting, sculpture, and photography.
- (8) Mathematical modeling in social sciences. The student applies mathematical processes to determine the number of elements in a finite sample space and compute the probability of an event. The student is expected to:
- (A) determine the number of ways an event such as a sports tournament may occur using combinations, permutations, and the Fundamental Counting Principle;
 - (B) compare theoretical to empirical probability such as determining if a particular game of chance is fair; and
 - (C) use experiments to determine the reasonableness of a theoretical model such as binomial or geometric.
- (9) Mathematical modeling in social sciences. The student applies mathematical processes and mathematical models to analyze data as it applies to social sciences. The student is expected to:
- (A) interpret information from various graphs, including line graphs, bar graphs, circle graphs, histograms, scatterplots, dot plots, stem-and-leaf plots, and box and whisker plots, to draw conclusions from the data and determine the strengths and weaknesses of conclusions;
 - (B) analyze numerical data using measures of central tendency (mean, median, and mode) and variability (range, interquartile range or IQR, and standard deviation) in order to make inferences with normal distributions;
 - (C) distinguish the purposes and differences among types of research, including surveys, experiments, and observational studies;
 - (D) use data from a sample to estimate population mean or population proportion;
 - (E) analyze marketing claims based on graphs and statistics from electronic and print media and justify the validity of stated or implied conclusions; and
 - (F) use regression methods available through technology to model linear and exponential functions, interpret correlations, and make predictions.
- (10) Mathematical modeling in social sciences. The student applies mathematical processes to design a study and use graphical, numerical, and analytical techniques to communicate the results of the study. The student is expected to:
- (A) formulate a meaningful question, determine the data needed to answer the question, gather the appropriate data, analyze the data, and draw reasonable conclusions; and
 - (B) communicate methods used, analyses conducted, and conclusions drawn for a data-analysis project through the use of one or more of the following: a written report, a visual display, an oral report, or a multi-media presentation.

§111.44. Advanced Quantitative Reasoning, Beginning with School Year 2013-2014 (One-Half to One Credit).

- (a) General requirements. Students shall be awarded one-half to one credit for successful completion of this course. Prerequisites: Geometry and Algebra II.
- (b) Introduction.

- (1) The desire to achieve educational excellence is the driving force behind the Texas essential knowledge and skills for mathematics, guided by the college and career readiness standards. By embedding statistics, probability, and finance, while focusing on fluency and solid understanding, Texas will lead the way in mathematics education and prepare all Texas students for the challenges they will face in the 21st century.
 - (2) The process standards describe ways in which students are expected to engage in the content. The placement of the process standards at the beginning of the knowledge and skills listed for each grade and course is intentional. The process standards weave the other knowledge and skills together so that students may be successful problem solvers and use mathematics efficiently and effectively in daily life. The process standards are integrated at every grade level and course. When possible, students will apply mathematics to problems arising in everyday life, society, and the workplace. Students will use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution. Students will select appropriate tools such as real objects, manipulatives, paper and pencil, and technology and techniques such as mental math, estimation, and number sense to solve problems. Students will effectively communicate mathematical ideas, reasoning, and their implications using multiple representations such as symbols, diagrams, graphs, and language. Students will use mathematical relationships to generate solutions and make connections and predictions. Students will analyze mathematical relationships to connect and communicate mathematical ideas. Students will display, explain, or justify mathematical ideas and arguments using precise mathematical language in written or oral communication.
 - (3) In Advanced Quantitative Reasoning, students will develop and apply skills necessary for college, careers, and life. Course content consists primarily of applications of high school mathematics concepts to prepare students to become well-educated and highly informed 21st century citizens. Students will develop and apply reasoning, planning, and communication to make decisions and solve problems in applied situations involving numerical reasoning, probability, statistical analysis, finance, mathematical selection, and modeling with algebra, geometry, trigonometry, and discrete mathematics.
 - (4) Statements that contain the word "including" reference content that must be mastered, while those containing the phrase "such as" are intended as possible illustrative examples.
- (c) Knowledge and skills.
- (1) Mathematical process standards. The student uses mathematical processes to acquire and demonstrate mathematical understanding. The student is expected to:
 - (A) apply mathematics to problems arising in everyday life, society, and the workplace;
 - (B) use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution;
 - (C) select tools, including real objects, manipulatives, paper and pencil, and technology as appropriate, and techniques, including mental math, estimation, and number sense as appropriate, to solve problems;
 - (D) communicate mathematical ideas, reasoning, and their implications using multiple representations, including symbols, diagrams, graphs, and language as appropriate;
 - (E) create and use representations to organize, record, and communicate mathematical ideas;
 - (F) analyze mathematical relationships to connect and communicate mathematical ideas; and
 - (G) display, explain, and justify mathematical ideas and arguments using precise mathematical language in written or oral communication.
 - (2) Numeric reasoning. The student applies the process standards in mathematics to generate new understandings by extending existing knowledge. The student generates new mathematical

understandings through problems involving numerical data that arise in everyday life, society, and the workplace. The student extends existing knowledge and skills to analyze real-world situations. The student is expected to:

- (A) use precision and accuracy in real-life situations related to measurement and significant figures;
 - (B) apply and analyze published ratings, weighted averages, and indices to make informed decisions;
 - (C) solve problems involving quantities that are not easily measured using proportionality such as packing problems, crowd estimation, and white blood cell count;
 - (D) solve geometric problems involving indirect measurement, including similar triangles, the Pythagorean Theorem, Law of Sines, Law of Cosines, and the use of dynamic geometry software;
 - (E) solve problems involving large quantities using combinatorics such as numbers of unique license plates and telephone numbers;
 - (F) use arrays to efficiently manage large collections of data and add, subtract, and multiply matrices to solve applied problems, including geometric transformations;
 - (G) analyze various voting and selection processes to compare results in given situations such as at-large versus single-member districts and plurality versus majority voting; and
 - (H) select and apply an algorithm of interest to solve real-life problems such as problems using recursion or iteration involving population growth or decline, fractals, and compound interest; the validity in recorded and transmitted data using checksums and hashing; sports rankings, weighted class rankings, and search engine rankings; and problems involving scheduling or routing situations using vertex-edge graphs, critical paths, Euler paths, and minimal spanning trees and communicate to peers the application of the algorithm in precise mathematical and nontechnical language.
- (3) Algebraic reasoning (expressions, equations, and generalized relationships). The student applies the process standards in mathematics to create and analyze mathematical models of everyday situations to make informed decisions related to earning, investing, spending, and borrowing money by appropriate, proficient, and efficient use of tools, including technology. The student uses mathematical relationships to make connections and predictions. The student judges the validity of a prediction and uses mathematical models to represent, analyze, and solve dynamic real-world problems. The student is expected to:
- (A) collect numerical bivariate data to create a scatterplot, select a function to model the data, justify the model selection, and use the model to interpret results and make predictions;
 - (B) describe the degree to which uncorrelated variables may or may not be related and analyze situations where correlated variables do or do not indicate a cause-and-effect relationship;
 - (C) determine or analyze an appropriate growth or decay model for problem situations, including linear, exponential, and logistic functions;
 - (D) determine or analyze an appropriate cyclical model for problem situations that can be modeled with periodic functions;
 - (E) determine or analyze an appropriate piecewise model for problem situations;
 - (F) create, represent, and analyze mathematical models for various types of income calculations to determine the best option for a given situation;
 - (G) create, represent, and analyze mathematical models for expenditures, including those involving credit, to determine the best option for a given situation; and

- (H) create, represent, and analyze mathematical models and appropriate representations, including formulas and amortization tables, for various types of loans and investments to determine the best option for a given situation such as cell phone plans and buying versus leasing a car.
- (4) Probabilistic and statistical reasoning. The student uses the process standards in mathematics to generate new understandings of probability and statistics. The student analyzes statistical information and evaluates risk and return to connect mathematical ideas and make informed decisions. The student applies a problem-solving model and statistical methods to design and conduct a study that addresses one or more particular question(s). The student uses multiple representations to communicate effectively the results of student-generated statistical studies and the critical analysis of published statistical studies. The student is expected to:
- (A) use a two-way frequency table as a sample space to identify whether two events are independent and to interpret the results;
- (B) use the Addition Rule, $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$, in mathematical and real-world problems;
- (C) calculate conditional probabilities and probabilities of compound events using tree diagrams, Venn diagrams, area models, and formulas such as Bayes' Theorem;
- (D) interpret conditional probabilities and probabilities of compound events by analyzing representations to make decisions in problem situations;
- (E) use probabilities to make and justify decisions about risks in everyday life such as the lottery, weather forecasts, and insurance costs;
- (F) calculate expected value to analyze mathematical fairness, payoff, and risk;
- (G) determine the validity of logical arguments that include compound conditional statements by constructing truth tables;
- (H) identify limitations and lack of relevant information in studies reporting statistical information, especially when studies are reported in condensed form;
- (I) interpret and compare statistical results using appropriate technology given a margin of error in situations such as polls, quality control, and measurements;
- (J) identify potential misuses of statistics to justify particular conclusions, including assertions of a cause-and-effect relationship rather than an association, and missteps or fallacies in logical reasoning such as confounding variables and hasty generalizations;
- (K) describe strengths and weaknesses of sampling techniques, data and graphical displays, and interpretations of summary statistics and other results appearing in a study, including reports published in the media;
- (L) determine the need for and purpose of a statistical investigation and what type of statistical analysis can be used to answer a specific question or set of questions;
- (M) identify the population of interest for a statistical investigation, select an appropriate sampling technique, and collect data;
- (N) identify the variables to be used in a study;
- (O) determine possible sources of statistical bias in a study and how bias may affect the validity of the results;
- (P) create data displays for given data sets to investigate, compare, and estimate center, shape, spread, and unusual features of the data;
- (Q) analyze possible sources of data variability, including those that can be controlled and those that cannot be controlled;

- (R) report results of statistical studies to a particular audience, including selecting an appropriate presentation format, creating graphical data displays, and interpreting results in terms of the question studied;
- (S) justify the design and the conclusion(s) of statistical studies, including the methods used; and
- (T) communicate statistical results in oral and written formats using appropriate statistical and nontechnical language.

§111.45. Independent Study in Mathematics, Beginning with School Year 2013-2014 (One-Half to One Credit).

(a) General requirements.

- (1) Students shall be awarded one-half to one credit for successful completion of this course. Prerequisites: Geometry and Algebra II.
- (2) Students may repeat this course with different course content for up to three credits.
- (3) The requirements for each course must be approved by the local district before the course begins.
- (4) If this course is being used to satisfy requirements for the Distinguished Achievement Program, student research/products must be presented before a panel of professionals or approved by the student's mentor.

(b) Introduction.

- (1) The desire to achieve educational excellence is the driving force behind the Texas essential knowledge and skills for mathematics, guided by the college and career readiness standards. By embedding statistics, probability, and finance, while focusing on fluency and solid understanding, Texas will lead the way in mathematics education and prepare all Texas students for the challenges they will face in the 21st century.
- (2) The process standards describe ways in which students are expected to engage in the content. The placement of the process standards at the beginning of the knowledge and skills listed for each grade and course is intentional. The process standards weave the other knowledge and skills together so that students may be successful problem solvers and use mathematics efficiently and effectively in daily life. The process standards are integrated at every grade level and course. When possible, students will apply mathematics to problems arising in everyday life, society, and the workplace. Students will use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution. Students will select appropriate tools such as real objects, manipulatives, paper and pencil, and technology and techniques such as mental math, estimation, and number sense to solve problems. Students will effectively communicate mathematical ideas, reasoning, and their implications using multiple representations such as symbols, diagrams, graphs, and language. Students will use mathematical relationships to generate solutions and make connections and predictions. Students will analyze mathematical relationships to connect and communicate mathematical ideas. Students will display, explain, or justify mathematical ideas and arguments using precise mathematical language in written or oral communication.
- (3) In Independent Study in Mathematics, students will extend their mathematical understanding beyond the Algebra II level in a specific area or areas of mathematics such as theory of equations, number theory, non-Euclidean geometry, linear algebra, advanced survey of mathematics, or history of mathematics.
- (4) Statements that contain the word "including" reference content that must be mastered, while those containing the phrase "such as" are intended as possible illustrative examples.

(c) Knowledge and skills: mathematical process standards. The student uses mathematical processes to acquire and demonstrate mathematical understanding. The student is expected to:

- (1) apply mathematics to problems arising in everyday life, society, and the workplace;
- (2) use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution;
- (3) select tools, including real objects, manipulatives, paper and pencil, and technology as appropriate, and techniques, including mental math, estimation, and number sense as appropriate, to solve problems;
- (4) communicate mathematical ideas, reasoning, and their implications using multiple representations, including symbols, diagrams, graphs, and language as appropriate;
- (5) create and use representations to organize, record, and communicate mathematical ideas;
- (6) analyze mathematical relationships to connect and communicate mathematical ideas; and
- (7) display, explain, and justify mathematical ideas and arguments using precise mathematical language in written or oral communication.

Subchapter D. Other High School Mathematics Courses

§111.51. Implementation of Texas Essential Knowledge and Skills for Mathematics, Other High School Mathematics Courses.

The provisions of this subchapter shall be implemented by school districts beginning September 1, 1998, and at that time shall supersede §75.63(o), (q)-(u), and (cc) of this title (relating to Mathematics).

§111.53. Advanced Placement (AP) Statistics (One-Half to One Credit).

- (a) General requirements. Students can be awarded one-half to one credit for successful completion of this course. Recommended prerequisites: Algebra II, Geometry.
- (b) Content requirements. Content requirements for Advanced Placement (AP) Statistics are prescribed in the College Board Publication Advanced Placement Course Description: Statistics, published by The College Board. This publication may be obtained from the College Board Advanced Placement Program.

§111.54. Advanced Placement (AP) Calculus AB (One-Half to One Credit).

- (a) General requirements. Students can be awarded one-half to one credit for successful completion of this course. Recommended prerequisite: Precalculus.
- (b) Content requirements. Content requirements for Advanced Placement (AP) Calculus AB are prescribed in the College Board Publication Advanced Placement Course Description Mathematics: Calculus AB, Calculus BC, published by The College Board. This publication may be obtained from the College Board Advanced Placement Program.

§111.55. Advanced Placement (AP) Calculus BC (One-Half to One Credit).

- (a) General requirements. Students can be awarded one-half to one credit for successful completion of this course. Recommended prerequisite: Precalculus.
- (b) Content requirements. Content requirements for Advanced Placement (AP) Calculus BC are prescribed in the College Board Publication Advanced Placement Course Description: Calculus AB, Calculus BC, published by The College Board. This publication may be obtained from the College Board Advanced Placement Program.

§111.56. IB Mathematical Studies Standard Level (One-Half to One Credit).

- (a) General requirements. Students can be awarded one-half to one credit for successful completion of IB Mathematical Studies Standard Level. To offer this course, the district must meet all requirements of the International Baccalaureate Organization, including teacher training/certification and IB assessment. Recommended prerequisites: Algebra II, Geometry.
- (b) Content requirements. Content requirements for IB Mathematical Studies Standard Level are prescribed by the International Baccalaureate Organization. Curriculum guides may be obtained from International Baccalaureate of North America.

§111.57. IB Mathematics Standard Level (One-Half to One Credit).

- (a) General requirements. Students can be awarded one-half to one credit for successful completion of IB Mathematics Standard Level. To offer this course, the district must meet all requirements of the International Baccalaureate Organization, including teacher training/certification and IB assessment. Recommended prerequisites: Algebra II, Geometry.
- (b) Content requirements. Content requirements for IB Mathematics Standard Level are prescribed by the International Baccalaureate Organization. Curriculum guides may be obtained from International Baccalaureate of North America.

§111.58. IB Mathematics Higher Level (One-Half to One Credit).

- (a) General requirements. Students can be awarded one-half to one credit for successful completion of IB Mathematics Higher Level. To offer this course, the district must meet all requirements of the International Baccalaureate Organization, including teacher training/certification and IB assessment. Recommended prerequisite: IB Mathematical Studies Standard Level or IB Mathematics Standard Level.
- (b) Content requirements. Content requirements for IB Mathematics Higher Level are prescribed by the International Baccalaureate Organization. Curriculum guides may be obtained from International Baccalaureate of North America.

§111.59. IB Further Mathematics Higher [Standard] Level (One-Half to One Credit).

- (a) General requirements. Students can be awarded one-half to one credit for successful completion of IB Further Mathematics Higher [Standard] Level. To offer this course, the district must meet all requirements of the International Baccalaureate Organization, including teacher training/certification and IB assessment. Recommended prerequisite: IB Mathematics Higher Level.
- (b) Content requirements. Content requirements for IB Further Mathematics Higher [Standard] Level are prescribed by the International Baccalaureate Organization. Curriculum guides may be obtained from International Baccalaureate of North America.

§111.60. Concurrent Enrollment in College Courses.

- (a) General requirements. Students shall be awarded at least one-half credit for each semester of successful completion of a college course in which the student is concurrently enrolled while in high school.
- (b) Content requirements. In order for students to receive state graduation credit for concurrent enrollment courses, content requirements must meet or exceed the essential knowledge and skills in a given course.

§111.61. Other Courses for Which Students May Receive Mathematics Credit.

- (a) Mathematical Applications in Agriculture, Food, and Natural Resources. Students on the minimum high school program or recommended high school program shall be awarded one credit in mathematics for successful completion of this course as described in §130.10 of this title (relating to Mathematical Applications in Agriculture, Food, and Natural Resources (One Credit)) and in accordance with the graduation requirements in Chapter 74 of this title (relating to Curriculum Requirements). Recommended prerequisite: a minimum of one credit from the courses in the Agriculture, Food, and Natural Resources cluster.
- (b) Engineering Mathematics. Students shall be awarded one credit in mathematics for successful completion of this course as described in §130.367 of this title (relating to Engineering Mathematics (One Credit)). Prerequisite: Algebra II. This course is recommended for students in Grades 11 and 12.
- (c) Statistics and Risk Management. Students shall be awarded one credit in mathematics for successful completion of this course as described in §130.169 of this title (relating to Statistics and Risk Management (One Credit)). Recommended prerequisites: Accounting I and Algebra II. This course is recommended for students in Grades 11 and 12.
- (d) Advanced Placement (AP) Computer Science A. Students may be awarded one mathematics credit for successful completion of this course. Content requirements for Advanced Placement (AP) Computer Science A are prescribed in the College Board Publication Advanced Placement Course Description: Computer Science A, published by The College Board. This publication may be obtained from the College Board Advanced Placement Program.