

**ATTACHMENT II**  
**Text of Proposed Amendments to 19 TAC**

**Chapter 111. Texas Essential Knowledge and Skills for Mathematics**

**Subchapter B. Middle School**

**§111.23. Mathematics, Grade 7.**

- (a) Introduction.
- (1) Within a well-balanced mathematics curriculum, the primary focal points at Grade 7 are using direct proportional relationships in number, geometry, measurement, and probability; applying addition, subtraction, multiplication, and division of decimals, fractions, and integers; and using statistical measures to describe data.
  - (2) Throughout mathematics in Grades 6-8, students build a foundation of basic understandings in number, operation, and quantitative reasoning; patterns, relationships, and algebraic thinking; geometry and spatial reasoning; measurement; and probability and statistics. Students use concepts, algorithms, and properties of rational numbers to explore mathematical relationships and to describe increasingly complex situations. Students use algebraic thinking to describe how a change in one quantity in a relationship results in a change in the other; and they connect verbal, numeric, graphic, and symbolic representations of relationships. Students use geometric properties and relationships, as well as spatial reasoning, to model and analyze situations and solve problems. Students communicate information about geometric figures or situations by quantifying attributes, generalize procedures from measurement experiences, and use the procedures to solve problems. Students use appropriate statistics, representations of data, reasoning, and concepts of probability to draw conclusions, evaluate arguments, and make recommendations.
  - (3) Problem solving in meaningful contexts, language and communication, connections within and outside mathematics, and formal and informal reasoning underlie all content areas in mathematics. Throughout mathematics in Grades 6-8, students use these processes together with graphing technology and other mathematical tools such as manipulative materials to develop conceptual understanding and solve problems as they do mathematics.
- (b) Knowledge and skills.
- (1) **Number, operation, and quantitative reasoning.** The student represents and uses numbers in a variety of equivalent forms. The student is expected to:
    - (A) compare and order integers and positive rational numbers;
    - (B) convert between fractions, decimals, whole numbers, and percents mentally, on paper, or with a calculator; and
    - (C) represent squares and square roots using geometric models.
  - (2) **Number, operation, and quantitative reasoning.** The student adds, subtracts, multiplies, or divides to solve problems and justify solutions. The student is expected to:
    - (A) represent multiplication and division situations involving fractions and decimals with models, including concrete objects, pictures, words, and numbers;
    - (B) use addition, subtraction, multiplication, and division to solve problems involving fractions and decimals;
    - (C) use models, such as concrete objects, pictorial models, and number lines, to add, subtract, multiply, and divide integers and connect the actions to algorithms;

- (D) use division to find unit rates and ratios in proportional relationships such as speed, density, price, recipes, and student-teacher ratio;
  - (E) simplify numerical expressions involving order of operations and exponents;
  - (F) select and use appropriate operations to solve problems and justify the selections; and
  - (G) determine the reasonableness of a solution to a problem.
- (3) **Patterns, relationships, and algebraic thinking.** The student solves problems involving direct proportional relationships. The student is expected to:
- (A) estimate and find solutions to application problems involving percent; and
  - (B) estimate and find solutions to application problems involving proportional relationships such as similarity, scaling, unit costs, and related measurement units.
- (4) **Patterns, relationships, and algebraic thinking.** The student represents a relationship in numerical, geometric, verbal, and symbolic form. The student is expected to:
- (A) generate formulas involving unit conversions within the same system (customary and metric), perimeter, area, circumference, volume, and scaling;
  - (B) graph data to demonstrate relationships in familiar concepts such as conversions, perimeter, area, circumference, volume, and scaling; and
  - (C) use words and symbols to describe the relationship between the terms in an arithmetic sequence (with a constant rate of change) and their positions in the sequence.
- (5) **Patterns, relationships, and algebraic thinking.** The student uses equations to solve problems. The student is expected to:
- (A) use concrete and pictorial models to solve equations and use symbols to record the actions; and
  - (B) formulate problem situations when given a simple equation and formulate an equation when given a problem situation.
- (6) **Geometry and spatial reasoning.** The student compares and classifies two- and three-dimensional figures using geometric vocabulary and properties. The student is expected to:
- (A) use angle measurements to classify pairs of angles as complementary or supplementary;
  - (B) use properties to classify triangles and quadrilaterals;
  - (C) use properties to classify three-dimensional figures, including pyramids, cones, prisms, and cylinders; and
  - (D) use critical attributes to define similarity.
- (7) **Geometry and spatial reasoning.** The student uses coordinate geometry to describe location on a plane. The student is expected to:
- (A) locate and name points on a coordinate plane using ordered pairs of integers; and
  - (B) graph reflections across the horizontal or vertical axis and graph translations on a coordinate plane.
- (8) **Geometry and spatial reasoning.** The student uses geometry to model and describe the physical world. The student is expected to:

- (A) sketch three-dimensional figures when given the top, side, and front views;
  - (B) make a net (two-dimensional model) of the surface area of a three-dimensional figure; and
  - (C) use geometric concepts and properties to solve problems in fields such as art and architecture.
- (9) **Measurement.** The student solves application problems involving estimation and measurement. The student is expected to:
- (A) estimate measurements and solve application problems involving length (including perimeter and circumference) and area of polygons and other shapes;
  - (B) connect models for volume of prisms (triangular and rectangular) and cylinders to formulas of prisms (triangular and rectangular) and cylinders; and
  - (C) estimate measurements and solve application problems involving volume of prisms (rectangular and triangular) and cylinders.
- (10) **Probability and statistics.** The student recognizes that a physical or mathematical model (including geometric) can be used to describe the experimental and theoretical probability of real-life events. The student is expected to:
- (A) construct sample spaces for simple or composite experiments; and
  - (B) find the probability of independent events.
- (11) **Probability and statistics.** The student understands that the way a set of data is displayed influences its interpretation. The student is expected to:
- (A) select and use an appropriate representation for presenting and displaying relationships among collected data, including line plot, line graph, bar graph, stem and leaf plot, circle graph, and Venn diagrams, and justify the selection; and
  - (B) make inferences and convincing arguments based on an analysis of given or collected data.
- (12) **Probability and statistics.** The student uses measures of central tendency and variability [range] to describe a set of data. The student is expected to:
- (A) describe a set of data using mean, median, mode, and range; and
  - (B) choose among mean, median, mode, or range to describe a set of data and justify the choice for a particular situation.
- (13) **Underlying processes and mathematical tools.** The student applies Grade 7 mathematics to solve problems connected to everyday experiences, investigations in other disciplines, and activities in and outside of school. The student is expected to:
- (A) identify and apply mathematics to everyday experiences, to activities in and outside of school, with other disciplines, and with other mathematical topics;
  - (B) use a problem-solving model that incorporates understanding the problem, making a plan, carrying out the plan, and evaluating the solution for reasonableness;
  - (C) select or develop an appropriate problem-solving strategy from a variety of different types, including drawing a picture, looking for a pattern, systematic guessing and checking, acting it out, making a table, working a simpler problem, or working backwards to solve a problem; and

- (D) select tools such as real objects, manipulatives, paper/pencil, and technology or techniques such as mental math, estimation, and number sense to solve problems.
- (14) **Underlying processes and mathematical tools.** The student communicates about Grade 7 mathematics through informal and mathematical language, representations, and models. The student is expected to:
- (A) communicate mathematical ideas using language, efficient tools, appropriate units, and graphical, numerical, physical, or algebraic mathematical models; and
  - (B) evaluate the effectiveness of different representations to communicate ideas.
- (15) **Underlying processes and mathematical tools.** The student uses logical reasoning to make conjectures and verify conclusions. The student is expected to:
- (A) make conjectures from patterns or sets of examples and nonexamples; and
  - (B) validate his/her conclusions using mathematical properties and relationships.

#### §111.24. Mathematics, Grade 8.

- (a) Introduction.
- (1) Within a well-balanced mathematics curriculum, the primary focal points at Grade 8 are using basic principles of algebra to analyze and represent both proportional and non-proportional linear relationships and using probability to describe data and make predictions.
  - (2) Throughout mathematics in Grades 6-8, students build a foundation of basic understandings in number, operation, and quantitative reasoning; patterns, relationships, and algebraic thinking; geometry and spatial reasoning; measurement; and probability and statistics. Students use concepts, algorithms, and properties of rational numbers to explore mathematical relationships and to describe increasingly complex situations. Students use algebraic thinking to describe how a change in one quantity in a relationship results in a change in the other; and they connect verbal, numeric, graphic, and symbolic representations of relationships. Students use geometric properties and relationships, as well as spatial reasoning, to model and analyze situations and solve problems. Students communicate information about geometric figures or situations by quantifying attributes, generalize procedures from measurement experiences, and use the procedures to solve problems. Students use appropriate statistics, representations of data, reasoning, and concepts of probability to draw conclusions, evaluate arguments, and make recommendations.
  - (3) Problem solving in meaningful contexts, language and communication, connections within and outside mathematics, and formal and informal reasoning underlie all content areas in mathematics. Throughout mathematics in Grades 6-8, students use these processes together with graphing technology and other mathematical tools such as manipulative materials to develop conceptual understanding and solve problems as they do mathematics.
- (b) Knowledge and skills.
- (1) **Number, operation, and quantitative reasoning.** The student understands that different forms of numbers are appropriate for different situations. The student is expected to:
    - (A) compare and order rational numbers in various forms including integers, percents, and positive and negative fractions and decimals;
    - (B) select and use appropriate forms of rational numbers to solve real-life problems including those involving proportional relationships;

- (C) approximate (mentally and with calculators) the value of irrational numbers as they arise from problem situations (such as  $\pi$ ,  $\sqrt{2}$ ); and
  - (D) express numbers in scientific notation, including negative exponents, in appropriate problem situations ; and [ $\pm$ ]
  - (E) compare and order real numbers with a calculator.
- (2) **Number, operation, and quantitative reasoning.** The student selects and uses appropriate operations to solve problems and justify solutions. The student is expected to:
- (A) select appropriate operations to solve problems involving rational numbers and justify the selections;
  - (B) use appropriate operations to solve problems involving rational numbers in problem situations;
  - (C) evaluate a solution for reasonableness; and
  - (D) use multiplication by a given constant factor (including unit rate) to represent and solve problems involving proportional relationships including conversions between measurement systems .
- (3) **Patterns, relationships, and algebraic thinking.** The student identifies proportional or non-proportional linear relationships in problem situations and solves problems. The student is expected to:
- (A) compare and contrast proportional and non-proportional linear relationships; and
  - (B) estimate and find solutions to application problems involving percents and other proportional relationships such as similarity and rates.
- (4) **Patterns, relationships, and algebraic thinking.** The student makes connections among various representations of a numerical relationship. The student is expected to generate a different representation of data given another representation of data (such as a table, graph, equation, or verbal description).
- (5) **Patterns, relationships, and algebraic thinking.** The student uses graphs, tables, and algebraic representations to make predictions and solve problems. The student is expected to:
- (A) predict, find, and justify solutions to application problems using appropriate tables, graphs, and algebraic equations; and
  - (B) find and evaluate an algebraic expression to determine any term in an arithmetic sequence (with a constant rate of change).
- (6) **Geometry and spatial reasoning.** The student uses transformational geometry to develop spatial sense. The student is expected to:
- (A) generate similar figures using dilations including enlargements and reductions; and
  - (B) graph dilations, reflections, and translations on a coordinate plane.
- (7) **Geometry and spatial reasoning.** The student uses geometry to model and describe the physical world. The student is expected to:
- (A) draw three-dimensional figures from different perspectives;
  - (B) use geometric concepts and properties to solve problems in fields such as art and architecture;
  - (C) use pictures or models to demonstrate the Pythagorean Theorem; and

- (D) locate and name points on a coordinate plane using ordered pairs of rational numbers.
- (8) **Measurement.** The student uses procedures to determine measures of three-dimensional figures. The student is expected to:
- (A) find lateral and total surface area of prisms, pyramids, and cylinders using concrete models and nets (two-dimensional models);
  - (B) connect models of prisms, cylinders, pyramids, spheres, and cones to formulas for volume of these objects; and
  - (C) estimate measurements and use formulas to solve application problems involving lateral and total surface area and volume.
- (9) **Measurement.** The student uses indirect measurement to solve problems. The student is expected to:
- (A) use the Pythagorean Theorem to solve real-life problems; and
  - (B) use proportional relationships in similar two-dimensional figures or similar three-dimensional figures to find missing measurements.
- (10) **Measurement.** The student describes how changes in dimensions affect linear, area, and volume measures. The student is expected to:
- (A) describe the resulting effects on perimeter and area when dimensions of a shape are changed proportionally; and
  - (B) describe the resulting effect on volume when dimensions of a solid are changed proportionally.
- (11) **Probability and statistics.** The student applies concepts of theoretical and experimental probability to make predictions. The student is expected to:
- (A) find the probabilities of dependent and independent events;
  - (B) use theoretical probabilities and experimental results to make predictions and decisions; and
  - (C) select and use different models to simulate an event.
- (12) **Probability and statistics.** The student uses statistical procedures to describe data. The student is expected to:
- (A) use variability (range, including interquartile range (IQR)) and select the appropriate measure of central tendency [~~or range~~] to describe a set of data and justify the choice for a particular situation;
  - (B) draw conclusions and make predictions by analyzing trends in scatterplots; and
  - (C) select and use an appropriate representation for presenting and displaying relationships among collected data, including line plots, line graphs, stem and leaf plots, circle graphs, bar graphs, box and whisker plots, histograms, and Venn diagrams, with and without the use of technology.
- (13) **Probability and statistics.** The student evaluates predictions and conclusions based on statistical data. The student is expected to:
- (A) evaluate methods of sampling to determine validity of an inference made from a set of data; and
  - (B) recognize misuses of graphical or numerical information and evaluate predictions and conclusions based on data analysis.

- (14) **Underlying processes and mathematical tools.** The student applies Grade 8 mathematics to solve problems connected to everyday experiences, investigations in other disciplines, and activities in and outside of school. The student is expected to:
- (A) identify and apply mathematics to everyday experiences, to activities in and outside of school, with other disciplines, and with other mathematical topics;
  - (B) use a problem-solving model that incorporates understanding the problem, making a plan, carrying out the plan, and evaluating the solution for reasonableness;
  - (C) select or develop an appropriate problem-solving strategy from a variety of different types, including drawing a picture, looking for a pattern, systematic guessing and checking, acting it out, making a table, working a simpler problem, or working backwards to solve a problem; and
  - (D) select tools such as real objects, manipulatives, paper/pencil, and technology or techniques such as mental math, estimation, and number sense to solve problems.
- (15) **Underlying processes and mathematical tools.** The student communicates about Grade 8 mathematics through informal and mathematical language, representations, and models. The student is expected to:
- (A) communicate mathematical ideas using language, efficient tools, appropriate units, and graphical, numerical, physical, or algebraic mathematical models; and
  - (B) evaluate the effectiveness of different representations to communicate ideas.
- (16) **Underlying processes and mathematical tools.** The student uses logical reasoning to make conjectures and verify conclusions. The student is expected to:
- (A) make conjectures from patterns or sets of examples and nonexamples; and
  - (B) validate his/her conclusions using mathematical properties and relationships.

## Subchapter C. High School

### §111.33. Algebra II (One-Half to One Credit).

- (a) Basic understandings.
- (1) Foundation concepts for high school mathematics. As presented in Grades K-8, the basic understandings of number, operation, and quantitative reasoning; patterns, relationships, and algebraic thinking; geometry; measurement; and probability and statistics are essential foundations for all work in high school mathematics. Students continue to build on this foundation as they expand their understanding through other mathematical experiences.
  - (2) Algebraic thinking and symbolic reasoning. Symbolic reasoning plays a critical role in algebra; symbols provide powerful ways to represent mathematical situations and to express generalizations. Students study algebraic concepts and the relationships among them to better understand the structure of algebra.
  - (3) Functions, equations, and their relationship. The study of functions, equations, and their relationship is central to all of mathematics. Students perceive functions and equations as means for analyzing and understanding a broad variety of relationships and as a useful tool for expressing generalizations.
  - (4) Relationship between algebra and geometry. Equations and functions are algebraic tools that can be used to represent geometric curves and figures; similarly, geometric figures can illustrate algebraic relationships. Students perceive the connections between algebra and geometry and use the tools of one to help solve problems in the other.
  - (5) Tools for algebraic thinking. Techniques for working with functions and equations are essential in understanding underlying relationships. Students use a variety of representations (concrete, pictorial, numerical, symbolic, graphical, and verbal), tools, and technology (including, but not limited to, calculators with graphing capabilities, data collection devices, and computers) to model mathematical situations to solve meaningful problems.
  - (6) Underlying mathematical processes. Many processes underlie all content areas in mathematics. As they do mathematics, students continually use problem-solving, language and communication, and reasoning (justification and proof) to make connections within and outside mathematics. Students also use multiple representations, technology, applications and modeling, and numerical fluency in problem-solving contexts.
- (b) Knowledge and skills.
- (1) **Foundations for functions.** The student uses properties and attributes of functions and applies functions to problem situations. The student is expected to:
    - (A) identify the mathematical domains and ranges of functions and determine reasonable domain and range values for continuous and discrete situations; and
    - (B) collect and organize data, make and interpret scatterplots, fit the graph of a function to the data, interpret the results, and proceed to model, predict, and make decisions and critical judgments.
  - (2) **Foundations for functions.** The student understands the importance of the skills required to manipulate symbols in order to solve problems and uses the necessary algebraic skills required to simplify algebraic expressions and solve equations and inequalities in problem situations. The student is expected to:
    - (A) use tools including factoring and properties of exponents to simplify expressions and to transform and solve equations; and

- (B) use complex numbers to describe the solutions of quadratic equations.
- (3) **Foundations for functions.** The student formulates systems of equations and inequalities from problem situations, uses a variety of methods to solve them, and analyzes the solutions in terms of the situations. The student is expected to:
- (A) analyze situations and formulate systems of equations in two or more unknowns or inequalities in two unknowns to solve problems;
- (B) use algebraic methods, graphs, tables, or matrices, to solve systems of equations or inequalities; and
- (C) interpret and determine the reasonableness of solutions to systems of equations or inequalities for given contexts.
- (4) **Algebra and geometry.** The student connects algebraic and geometric representations of functions. The student is expected to:
- (A) identify and sketch graphs of parent functions, including linear ( $f(x) = x$ ), quadratic ( $f(x) = x^2$ ), exponential ( $f(x) = a^x$ ), and logarithmic ( $f(x) = \log_a x$ ) functions, absolute value of  $x$  ( $f(x) = |x|$ ), square root of  $x$  ( $f(x) = \sqrt{x}$ ), and reciprocal of  $x$  ( $f(x) = 1/x$ );
- (B) extend parent functions with parameters such as  $a$  in  $f(x) = a/x$  and describe the effects of the parameter changes on the graph of parent functions; and
- (C) describe and analyze the relationship between a function and its inverse.
- (5) **Algebra and geometry.** The student knows the relationship between the geometric and algebraic descriptions of conic sections. The student is expected to:
- (A) describe a conic section as the intersection of a plane and a cone;
- (B) sketch graphs of conic sections to relate simple parameter changes in the equation to corresponding changes in the graph;
- (C) identify symmetries from graphs of conic sections;
- (D) identify the conic section from a given equation; and
- (E) use the method of completing the square.
- (6) **Quadratic and square root functions.** The student understands that quadratic functions can be represented in different ways and translates among their various representations. The student is expected to:
- (A) determine the reasonable domain and range values of quadratic functions, as well as interpret and determine the reasonableness of solutions to quadratic equations and inequalities;
- (B) relate representations of quadratic functions, such as algebraic, tabular, graphical, and verbal descriptions; and
- (C) determine a quadratic function from its roots (real and complex) or a graph.
- (7) **Quadratic and square root functions.** The student interprets and describes the effects of changes in the parameters of quadratic functions in applied and mathematical situations. The student is expected to:
- (A) use characteristics of the quadratic parent function to sketch the related graphs and connect between the  $y = ax^2 + bx + c$  and the

$y = a(x - h)^2 + k$  symbolic representations of quadratic functions; and

- (B) use the parent function to investigate, describe, and predict the effects of changes in  $a$ ,  $h$ , and  $k$  on the graphs of  $y = a(x - h)^2 + k$  form of a function in applied and purely mathematical situations.
- (8) **Quadratic and square root functions.** The student formulates equations and inequalities based on quadratic functions, uses a variety of methods to solve them, and analyzes the solutions in terms of the situation. The student is expected to:
- (A) analyze situations involving quadratic functions and formulate quadratic equations or inequalities to solve problems;
  - (B) analyze and interpret the solutions of quadratic equations using discriminants and solve quadratic equations using the quadratic formula;
  - (C) compare and translate between algebraic and graphical solutions of quadratic equations; and
  - (D) solve quadratic equations and inequalities using graphs, tables, and algebraic methods.
- (9) **Quadratic and square root functions.** The student formulates equations and inequalities based on square root functions, uses a variety of methods to solve them, and analyzes the solutions in terms of the situation. The student is expected to:
- (A) use the parent function to investigate, describe, and predict the effects of parameter changes on the graphs of square root functions and describe limitations on the domains and ranges;
  - (B) relate representations of square root functions, such as algebraic, tabular, graphical, and verbal descriptions;
  - (C) determine the reasonable domain and range values of square root functions, as well as interpret and determine the reasonableness of solutions to square root equations and inequalities;
  - (D) determine solutions of square root equations using graphs, tables, and algebraic methods;
  - (E) determine solutions of square root inequalities using graphs and tables;
  - (F) analyze situations modeled by square root functions, formulate equations or inequalities, select a method, and solve problems; and
  - (G) connect inverses of square root functions with quadratic functions.
- (10) **Rational functions.** The student formulates equations and inequalities based on rational functions, uses a variety of methods to solve them, and analyzes the solutions in terms of the situation. The student is expected to:
- (A) use quotients of polynomials to describe the graphs of rational functions, predict the effects of parameter changes, describe limitations on the domains and ranges, and examine asymptotic behavior;
  - (B) analyze various representations of rational functions with respect to problem situations;
  - (C) determine the reasonable domain and range values of rational functions, as well as interpret and determine the reasonableness of solutions to rational equations and inequalities;
  - (D) determine the solutions of rational equations using graphs, tables, and algebraic methods;

- (E) determine solutions of rational inequalities using graphs and tables;
  - (F) analyze a situation modeled by a rational function, formulate an equation or inequality composed of a linear or quadratic function, and solve the problem; and
  - (G) use functions to model and make predictions in problem situations involving direct and inverse variation.
- (11) **Exponential and logarithmic functions.** The student formulates equations and inequalities based on exponential and logarithmic functions, uses a variety of methods to solve them, and analyzes the solutions in terms of the situation. The student is expected to:
- (A) develop the definition of logarithms by exploring and describing the relationship between exponential functions and their inverses;
  - (B) use the parent functions to investigate, describe, and predict the effects of parameter changes on the graphs of exponential and logarithmic functions, describe limitations on the domains and ranges, and examine asymptotic behavior;
  - (C) determine the reasonable domain and range values of exponential and logarithmic functions, as well as interpret and determine the reasonableness of solutions to exponential and logarithmic equations and inequalities;
  - (D) determine solutions of exponential and logarithmic equations using graphs, tables, and algebraic methods;
  - (E) determine solutions of exponential and logarithmic inequalities using graphs and tables; and
  - (F) analyze a situation modeled by an exponential function, formulate an equation or inequality, and solve the problem.

**§111.34. Geometry (One Credit).**

- (a) Basic understandings.
  - (1) Foundation concepts for high school mathematics. As presented in Grades K-8, the basic understandings of number, operation, and quantitative reasoning; patterns, relationships, and algebraic thinking; geometry; measurement; and probability and statistics are essential foundations for all work in high school mathematics. Students continue to build on this foundation as they expand their understanding through other mathematical experiences.
  - (2) Geometric thinking and spatial reasoning. Spatial reasoning plays a critical role in geometry; geometric figures provide powerful ways to represent mathematical situations and to express generalizations about space and spatial relationships. Students use geometric thinking to understand mathematical concepts and the relationships among them.
  - (3) Geometric figures and their properties. Geometry consists of the study of geometric figures of zero, one, two, and three dimensions and the relationships among them. Students study properties and relationships having to do with size, shape, location, direction, and orientation of these figures.
  - (4) The relationship between geometry, other mathematics, and other disciplines. Geometry can be used to model and represent many mathematical and real-world situations. Students perceive the connection between geometry and the real and mathematical worlds and use geometric ideas, relationships, and properties to solve problems.

- (5) Tools for geometric thinking. Techniques for working with spatial figures and their properties are essential in understanding underlying relationships. Students use a variety of representations (concrete, pictorial, numerical, symbolic, graphical, and verbal), tools, and technology (including, but not limited to, calculators with graphing capabilities, data collection devices, and computers) to solve meaningful problems by representing and transforming figures and analyzing relationships.
- (6) Underlying mathematical processes. Many processes underlie all content areas in mathematics. As they do mathematics, students continually use problem-solving, language and communication, connections within and outside mathematics, and reasoning (justification and proof). Students also use multiple representations, technology, applications and modeling, and numerical fluency in problem solving contexts.
- (b) Knowledge and skills.
- (1) **Geometric structure.** The student understands the structure of, and relationships within, an axiomatic system. The student is expected to:
- (A) develop an awareness of the structure of a mathematical system, connecting definitions, postulates, logical reasoning, and theorems;
  - (B) recognize the historical development of geometric systems and know mathematics is developed for a variety of purposes; and
  - (C) compare and contrast the structures and implications of Euclidean and non-Euclidean geometries.
- (2) **Geometric structure.** The student analyzes geometric relationships in order to make and verify conjectures. The student is expected to:
- (A) use constructions to explore attributes of geometric figures and to make conjectures about geometric relationships; and
  - (B) make conjectures about angles, lines, polygons, circles, and three-dimensional figures and determine the validity of the conjectures, choosing from a variety of approaches such as coordinate, transformational, or axiomatic.
- (3) **Geometric structure.** The student applies logical reasoning to justify and prove mathematical statements. The student is expected to:
- (A) determine the validity of a conditional statement, its converse, inverse, and contrapositive;
  - (B) construct and justify statements about geometric figures and their properties;
  - (C) use logical reasoning to prove statements are true and find counter examples to disprove statements that are false;
  - (D) use inductive reasoning to formulate a conjecture; and
  - (E) use deductive reasoning to prove a statement.
- (4) **Geometric structure.** The student uses a variety of representations to describe geometric relationships and solve problems. The student is expected to select an appropriate representation (concrete, pictorial, graphical, verbal, or symbolic) in order to solve problems.
- (5) **Geometric patterns.** The student uses a variety of representations to describe geometric relationships and solve problems. The student is expected to:
- (A) use numeric and geometric patterns to develop algebraic expressions representing geometric properties;

- (B) use numeric and geometric patterns to make generalizations about geometric properties, including properties of polygons, ratios in similar figures and solids, and angle relationships in polygons and circles;
  - (C) use properties of transformations and their compositions to make connections between mathematics and the real world, such as tessellations; and
  - (D) identify and apply patterns from right triangles to solve meaningful problems, including special right triangles (45-45-90 and 30-60-90) and triangles whose sides are Pythagorean triples.
- (6) **Dimensionality and the geometry of location.** The student analyzes the relationship between three-dimensional geometric figures and related two-dimensional representations and uses these representations to solve problems. The student is expected to:
- (A) describe and draw the intersection of a given plane with various three-dimensional geometric figures;
  - (B) use nets to represent and construct three-dimensional geometric figures; and
  - (C) use orthographic and isometric views of three-dimensional geometric figures to represent and construct three-dimensional geometric figures and solve problems.
- (7) **Dimensionality and the geometry of location.** The student understands that coordinate systems provide convenient and efficient ways of representing geometric figures and uses them accordingly. The student is expected to:
- (A) use one- and two-dimensional coordinate systems to represent points, lines, rays, line segments, and figures;
  - (B) use slopes and equations of lines to investigate geometric relationships, including parallel lines, perpendicular lines, and special segments of triangles and other polygons; and
  - (C) derive and use formulas involving length, slope, and midpoint.
- (8) **Congruence and the geometry of size.** The student uses tools to determine measurements of geometric figures and extends measurement concepts to find perimeter, area, and volume in problem situations. The student is expected to:
- (A) find areas of regular polygons, circles, and composite figures;
  - (B) find areas of sectors and arc lengths of circles using proportional reasoning;
  - (C) derive, extend, and use the Pythagorean Theorem; ~~and~~
  - (D) find surface areas and volumes of prisms, pyramids, spheres, cones, cylinders, and composites of these figures in problem situations ; and [ ]
  - (E) use area models to connect geometry to probability and statistics; and
  - (F) use conversions between measurement systems to solve problems in real-world situations.
- (9) **Congruence and the geometry of size.** The student analyzes properties and describes relationships in geometric figures. The student is expected to:
- (A) formulate and test conjectures about the properties of parallel and perpendicular lines based on explorations and concrete models;
  - (B) formulate and test conjectures about the properties and attributes of polygons and their component parts based on explorations and concrete models;
  - (C) formulate and test conjectures about the properties and attributes of circles and the lines that intersect them based on explorations and concrete models; and

- (D) analyze the characteristics of polyhedra and other three-dimensional figures and their component parts based on explorations and concrete models.
- (10) **Congruence and the geometry of size.** The student applies the concept of congruence to justify properties of figures and solve problems. The student is expected to:
  - (A) use congruence transformations to make conjectures and justify properties of geometric figures including figures represented on a coordinate plane; and
  - (B) justify and apply triangle congruence relationships.
- (11) **Similarity and the geometry of shape.** The student applies the concepts of similarity to justify properties of figures and solve problems. The student is expected to:
  - (A) use and extend similarity properties and transformations to explore and justify conjectures about geometric figures;
  - (B) use ratios to solve problems involving similar figures;
  - (C) develop, apply, and justify triangle similarity relationships, such as right triangle ratios, trigonometric ratios, and Pythagorean triples using a variety of methods; and
  - (D) describe the effect on perimeter, area, and volume when one or more dimensions of a figure are changed and apply this idea in solving problems.

**§111.36. Mathematical Models with Applications (One-Half to One Credit).**

- (a) General requirements. The provisions of this section shall be implemented beginning September 1, 1998. Students can be awarded one-half to one credit for successful completion of this course. Recommended prerequisite: Algebra I.
- (b) Introduction.
  - (1) In Mathematical Models with Applications, students continue to build on the K-8 and Algebra I foundations as they expand their understanding through other mathematical experiences. Students use algebraic, graphical, and geometric reasoning to recognize patterns and structure, to model information, and to solve problems from various disciplines. Students use mathematical methods to model and solve real-life applied problems involving money, data, chance, patterns, music, design, and science. Students use mathematical models from algebra, geometry, probability, and statistics and connections among these to solve problems from a wide variety of advanced applications in both mathematical and nonmathematical situations. Students use a variety of representations (concrete, pictorial, numerical, symbolic, graphical, and verbal), tools, and technology (including, but not limited to, calculators with graphing capabilities, data collection devices, and computers) to link modeling techniques and purely mathematical concepts and to solve applied problems.
  - (2) As students do mathematics, they continually use problem-solving, language and communication, connections within and outside mathematics, and reasoning (justification and proof). Students also use multiple representations, technology, applications and modeling, and numerical fluency in problem-solving contexts.
- (c) Knowledge and skills.
  - (1) The student uses a variety of strategies and approaches to solve both routine and non-routine problems. The student is expected to:
    - (A) compare and analyze various methods for solving a real-life problem;
    - (B) use multiple approaches (algebraic, graphical, and geometric methods) to solve problems from a variety of disciplines; and

- (C) select a method to solve a problem, defend the method, and justify the reasonableness of the results.
- (2) The student uses graphical and numerical techniques to study patterns and analyze data. The student is expected to:
- (A) interpret information from various graphs, including line graphs, bar graphs, circle graphs, histograms, scatterplots, line plots, stem and leaf plots, and box and whisker plots to draw conclusions from the data;
  - (B) analyze numerical data using measures of central tendency, variability, and correlation in order to make inferences;
  - (C) analyze graphs from journals, newspapers, and other sources to determine the validity of stated arguments; and
  - (D) use regression methods available through technology to describe various models for data such as linear, quadratic, exponential, etc., select the most appropriate model, and use the model to interpret information.
- (3) The student develops and implements a plan for collecting and analyzing data (qualitative and quantitative) in order to make decisions. The student is expected to:
- (A) formulate a meaningful question, determine the data needed to answer the question, gather the appropriate data, analyze the data, and draw reasonable conclusions;
  - (B) communicate methods used, analyses conducted, and conclusions drawn for a data-analysis project by written report, visual display, oral report, or multi-media presentation; and
  - (C) determine the appropriateness of a model for making predictions from a given set of data.
- (4) The student uses probability models to describe everyday situations involving chance. The student is expected to:
- (A) compare theoretical and empirical probability; and
  - (B) use experiments to determine the reasonableness of a theoretical model such as binomial, geometric, etc.
- (5) The student uses functional relationships to solve problems related to personal income. The student is expected to:
- (A) use rates, linear functions, and direct variation to solve problems involving personal finance and budgeting, including compensations and deductions;
  - (B) solve problems involving personal taxes; and
  - (C) analyze data to make decisions about banking.
- (6) The student uses algebraic formulas, graphs, and amortization models to solve problems involving credit. The student is expected to:
- (A) analyze methods of payment available in retail purchasing and compare relative advantages and disadvantages of each option;
  - (B) use amortization models to investigate home financing and compare buying and renting a home; and
  - (C) use amortization models to investigate automobile financing and compare buying and leasing a vehicle.
- (7) The student uses algebraic formulas, numerical techniques, and graphs to solve problems related to financial planning. The student is expected to:

- (A) analyze types of savings options involving simple and compound interest and compare relative advantages of these options;
  - (B) analyze and compare coverage options and rates in insurance; and
  - (C) investigate and compare investment options including stocks, bonds, annuities, and retirement plans.
- (8) The student uses algebraic and geometric models to describe situations and solve problems. The student is expected to:
- (A) use geometric models available through technology to model growth and decay in areas such as population, biology, and ecology;
  - (B) use trigonometric ratios and functions available through technology to calculate distances and model periodic motion; and
  - (C) use direct and inverse variation to describe physical laws such as Hook's, Newton's, and Boyle's laws.
- (9) The student uses algebraic and geometric models to represent patterns and structures. The student is expected to:
- (A) use geometric transformations, symmetry, and perspective drawings to describe mathematical patterns and structure in art and architecture; and
  - (B) use geometric transformations, proportions, and periodic motion to describe mathematical patterns and structure in music.